

SOME FURTHER CHAPTER MATERIALS

PART ONE

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CHAPTER 1. MOTION MATTERS

Instantaneous Speed

In Section 1.3 we discuss the average speed of an object, which is defined to be the ratio of the distance traveled, Δd , divided by the time interval, Δt :

$$v_{\text{av}} = \frac{\Delta d}{\Delta t}.$$

You may have noticed that we cannot measure the speed of an object in an instant of time. The average speed is the only kind of speed that we can actually measure, since we can only measure distance *intervals* and time *intervals*. We can use more sophisticated instruments to obtain the distance traveled in smaller and smaller time intervals. If the time interval Δt has approached zero, we are dealing with an *instant in time*, and the average speed

becomes the actual speed at that instant. This is called the *instantaneous speed*. However, in any real experiment we can never actually achieve an instant in time, an infinitesimally small time interval, since every measurement, no matter how fast we can make it, still takes some amount of time.

Nevertheless, we can use a graph of the motion to *calculate* a reasonable value for the instantaneous speed at an instant of time. We point out in Section 1.4 that the slope of the line on a distance–time graph is

$$\text{slope of line} = \frac{\Delta d}{\Delta t},$$

which is just the average speed during the time interval Δt . As the time interval becomes smaller and smaller, the line on the graph during the time interval becomes straighter and straighter. In such a situation, for very tiny time intervals, the average speed becomes, by definition, equal to the instantaneous speed at the center of the time interval. To put it differently, as the value of Δt approaches the limit of zero (which we cannot actually measure), the value for the average speed v_{av} approaches the instantaneous speed, which is given the symbol v . In this case, the slope of the line becomes a tangent to the curve at that instant. This means: *the instantaneous speed of an object at an instant of time t is defined as the tangent at time t to the line representing the object's motion on a distance–time graph.*

This can also be expressed in mathematical symbols as follows:

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta d}{\Delta t} = v.$$

In words, this says that in the limit as the time interval approaches zero, the ratio of the distance traveled divided by the time interval approaches the instantaneous speed at the time t at the center of the original time interval. (Readers who have had some calculus may recognize this as a differential.)

Derivation of Galileo's Expression $d = 1/2at^2$

Galileo's famous expression gives the distance (d) traveled by an object starting from rest and moving with uniform acceleration (a) during the time interval (t). Note that this expression does not contain the speed, only the distance and time, starting from zero, and the acceleration.

Galileo originally used a geometrical argument to derive this expression. Algebra was used more than 100 years later to derive the same expression.

Since it is more straightforward, we will use the algebraic derivation, along with some of Galileo's original assumptions.

We start with the definition of the average speed of a uniformly accelerating object during the time interval Δt . (This expression holds no matter how the object is moving.)

$$v_{\text{av}} = \frac{\Delta d}{\Delta t}.$$

We can rewrite this equation as

$$\Delta d = v_{\text{av}} \times \Delta t.$$

What would be the average velocity for a uniformly accelerating object? Galileo reasoned (as others had before him) that for any quantity that changes uniformly, the average value is just halfway between the beginning value and the final value. For uniformly accelerated motion starting from rest, the initial speed is zero, $v_{\text{initial}} = 0$. So, the average speed is halfway between 0 and v_{final} :

$$v_{\text{av}} = 1/2 v_{\text{final}}.$$

Substituting, we have

$$\Delta d = 1/2 v_{\text{final}} \times \Delta t.$$

Now we have to obtain a value for v_{final} . We can do this by starting with Galileo's *definition* of average acceleration

$$a_{\text{av}} = \frac{\Delta v}{\Delta t}.$$

In our case, a_{av} has a constant value, a , since the acceleration is uniform (constant). The value of Δv is $v_{\text{final}} - v_{\text{initial}}$, which is just v_{final} , since $v_{\text{initial}} = 0$. Substituting in the equation for a_{av} , we have

$$a = \frac{v_{\text{final}}}{\Delta t}.$$

Rearranging, we get

$$v_{\text{final}} = a \times \Delta t.$$

So now we can replace v_{final} in the expression for Δd , and we obtain

$$\Delta d = \frac{1}{2}v_{\text{final}} \times \Delta t,$$

$$\Delta d = \frac{1}{2}a(\Delta t)^2.$$

This is equivalent to Galileo's expression. If we measure the distance and the time interval from the position and the instant when the motion starts, then d_{initial} and t_{initial} are zero. We can then write this equation as

$$d_{\text{final}} = \frac{1}{2}at_{\text{final}}^2.$$

Or, if we let $d_{\text{final}} = d$ and $t_{\text{final}} = t$, we have an even simpler expression

$$d = \frac{1}{2}at^2.$$

If we start with a nonzero initial speed, then we have

$$d = v_{\text{initial}} t + \frac{1}{2}at^2.$$

CHAPTER 3. UNDERSTANDING MOTION

Derivation of the Parabolic Trajectory of a Projectile

The motion of a projectile is composed of two independent motions: uniform velocity in the horizontal direction and uniform acceleration in the vertical direction. During the time interval t , the distance traveled by the projectile in the horizontal direction, x , with uniform speed v_x is

$$x = v_x t.$$

The distance the projectile moves in the vertical direction, y , during the same time interval t is

$$y = \frac{1}{2}gt^2.$$

Solving the equation $x = v_x t$ for t gives

$$t = \frac{x}{v_x}.$$

Because the time interval t is the same in both equations, we can substitute x/v_x for t in the equation for y . This gives

$$y = \frac{1}{2}g\left(\frac{x}{v_x}\right)^2,$$

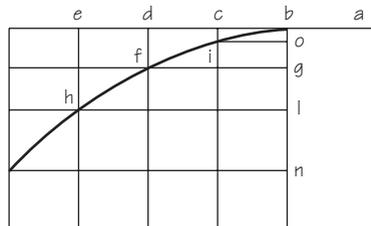
or

$$y = \left(\frac{g}{2v_x^2}\right)x^2.$$

This last equation contains two variables, x and y . It also contains three constant quantities: g , 2 , and the horizontal speed v_x . The vertical distance y that the projectile falls is thus a constant times the square of the horizontal displacement x :

$$y = (\text{constant})x^2.$$

The mathematical curve represented by this relationship between x and y is called a parabola. Galileo deduced the parabolic shape of trajectories by an argument similar to the one used here. This discovery greatly simplified the study of projectile motion, because the geometry of the parabola had been established centuries earlier by Greek mathematicians.



Drawing of a parabolic trajectory from Galileo's *Two New Sciences*.

Derivation of the Equation for Centripetal Acceleration, $a_c = v^2/R$

Assume that a stone on the end of a string is moving uniformly in a circle of radius R . You can find the relationship between a_c , v , and R by treating a small part of the circular path as a combination of tangential motion and acceleration toward the center. To follow the circular path, the stone must

accelerate toward the center through a distance b in the *same time* that it would move through a tangential distance d . The stone, with speed v , would travel a tangential distance d given by $d = v\Delta t$. In the same time Δt , the stone, with acceleration a_c , would travel toward the center through a distance b given by $b = \frac{1}{2}a_c \Delta t^2$. (You can use this last equation because at $t = 0$, the stone's velocity toward the center is zero.)

You can apply the Pythagorean theorem to the triangle in the figure that follows:

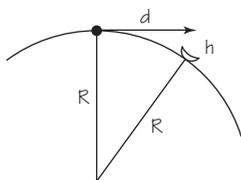
$$\begin{aligned} R^2 - d^2 &= (R+b)^2 \\ &= R^2 + 2Rb + b^2. \end{aligned}$$

When you subtract R^2 from each side of the equation, you are left with

$$d^2 = 2Rb + b^2.$$

You can simplify this expression by making an approximation. Since b is very small compared to R , b^2 will be very small compared to Rb . And since Δt must be vanishingly small to get the instantaneous acceleration, b^2 will become vanishingly small compared to Rb . So you can neglect b^2 and write

$$d^2 = 2Rb.$$



Also, $d = v\Delta t$ and $b = \frac{1}{2}a_c\Delta t^2$; so you can substitute for d^2 and for b accordingly. Thus,

$$(v \Delta t)^2 = 2R \cdot \frac{1}{2}a_c (\Delta t)^2,$$

$$v^2(\Delta t)^2 = Ra_c(\Delta t)^2,$$

$$v^2 = Ra_c,$$

or

$$a_c = \frac{v^2}{R}.$$

The approximation becomes better and better as Δt becomes smaller and smaller. In other words, v^2/R gives the magnitude of the *instantaneous* centripetal acceleration for a body moving on a circular arc of radius R . For uniform circular motion, v^2/R gives the magnitude of the centripetal acceleration at every point of the path. (Of course, it does not have to be a stone on a string. It can be a small particle on the rim of a rotating wheel, or a house on the rotating Earth, or a coin sitting on a rotating phonograph disk, or a car in a curve on the road, an electron in its path through a magnetic field, or the Moon going around the Earth in a nearly circular path.)

The relationship among a_c , v , and R was discovered by the Dutch scientists Christiaan Huygens and was published by him in 1673. Newton, however, must have known it in 1666, but he did not publish his proof until 1687, in the *Principia*.

We can substitute the relation $v = 2\pi Rf$ or $v = 2\pi R/T$ (see Section 4.11) into the equation for a_c :

$$\begin{aligned} a_c &= \frac{v^2}{R} \\ &= \frac{(2\pi Rf)^2}{R} \\ &= 4\pi^2 Rf^2 \end{aligned}$$

or

$$a_c = \frac{4\pi^2 R}{T^2}.$$

These two resulting expressions for a_c are entirely equivalent.

CHAPTER 4. NEWTON'S UNIFIED THEORY

"Weighing the Earth"

Now that we know how g arises in terms of Newton's law of universal gravitation, we can use the last equation above to find the mass of the Earth. This is possible because all of the terms in this equation are known, except

for M_{Earth} . To find M_{Earth} , first solve for it in the equation, using simple algebra:

$$M_{\text{Earth}} = \frac{gR_{\text{Earth}}}{G}.$$

(Be sure that you understand each step in obtaining the answer below; look at the review of scientific notation in the *Student Guide*, if necessary.)

Now substitute the known values on the right side of the equation

$$M_{\text{Earth}} = \frac{(9.8 \text{ m/s}^2)(6.4 \times 10^6 \text{ m})^2}{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)}.$$

To obtain a result from this expression, we perform all of the indicated arithmetic on the numbers and, separately, on the units. We'll first collect each of these together, which results in the following:

$$M_{\text{Earth}} = \frac{(9.8)(6.4 \times 10^6)^2}{6.67 \times 10^{-11}} \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

For simplicity, let's first work on the numbers (but never forgetting the units, which we'll carry along). We start by squaring the term in the numerator

$$(6.4 \times 10^6)^2 = 40.96 \times 10^{12} \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

So now we have

$$\frac{(9.8)(40.96 \times 10^{12})}{6.67 \times 10^{-11}} \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}.$$

Multiply and divide the numbers, then subtract the exponent of the denominator from that of the numerator

$$\begin{aligned} M_{\text{Earth}} &= \frac{(9.8)(40.96)}{6.67} \times 10^{12} \times 10^{11} \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2} \\ &= 60.18 \times 10^{23} = 6.02 \times 10^{24} \frac{(\text{m/s}^2)(\text{m})^2}{\text{N m}^2/\text{kg}^2}. \end{aligned}$$

Now let's work on the units (carrying along the numerical value):

$$\begin{aligned} M_{\text{Earth}} &= 6.02 \times 10^{24} \frac{(\text{m})(\text{m})^2 (\text{kg})^2}{(\text{s}^2)(\text{Nm}^2)} \\ &= 6.02 \times 10^{24} \frac{\text{m}^3 \text{kg}^2}{\text{s}^2 \text{Nm}^2}. \end{aligned}$$

Cancel the m^2 :

$$= 6.02 \times 10^{24} \frac{\text{m kg}^2}{\text{s}^2 \text{N}}.$$

By definition $1 \text{ N} = 1 \text{ kg m/s}^2$. Substituting for N we have

$$= 6.02 \times 10^{24} \frac{\text{m kg}^2 \text{ s}^2}{\text{s}^2 \text{ kg m}}.$$

Canceling as indicated, we are left simply with kg. So our final result is

$$M_{\text{Earth}} = 6.02 \times 10^{24} \text{ kg}.$$

This is a lot of mass, and the Earth is only one small blue planet just 4000 miles in radius! The value we have obtained agrees with the mass of the Earth obtained by other means, once again confirming Newton's theory.

Newton's Work: Impact and Reaction

Newton's work opened whole new lines of investigation, both theoretical and observational. In fact, much of our present science and also our technology had their effective beginnings with the work of Newton and those who followed in his spirit. New models, new mathematical tools, and a new confidence encouraged those followers to attack new problems, to open new vistas of research, and to answer long-standing questions. The modern view of science is that it is a continuing exploration of ever more interesting fields.

Newton's influence was not limited to science alone. The period following his death in 1727 was a period of further understanding and application of his discoveries and method. His influence was felt especially in

philosophy and literature, but also in many other fields outside science. Let us round out our view of Newton by referring to some of these effects.

The eighteenth century is often called the Age of Reason, the apogee of the so-called Enlightenment. “Reason” was the motto of the eighteenth-century philosophers. Enlightened by reason, especially scientific reason, humanity would overcome the darkness of ignorance and usher in a new age of the flowering of human potential. Such ideals appeared, for instance, in the following excerpt from *Hymn to Science* by the poet Mark Akenside (1721–1770).*

Science! thou fair effusive ray
 From the great source of mental day,
 Free, generous, and refined!
 Descend with all thy treasures fraught,
 Illuminate each bewilder'd thought,
 And bless my labouring mind. . . .

Oh! let thy powerful charms impart
 The patient head, the candid heart,
 Devoted to thy sway;
 Which no weak passions e'er mislead,
 Which still with dauntless steps proceed
 Where reason points the way. . . .

Give me to learn each secret cause;
 Let Number's, Figure's, Motion's laws
 Reveal'd before me stand;
 These to great Nature's scenes apply,
 And round the globe, and through the sky,
 Disclose her working hand.

Many thinkers of the Enlightenment believed they could extend the triumph of human reason in science to other areas of human endeavor. As a result, Newtonian physics, religious toleration, and republican government were all advanced by the same movement. However, their theories about improving religion and society were not convincingly connected. This does not mean there was really a logical link among these concepts. Nor were many eighteenth-century thinkers, in any field or nation, much bothered by other gaps in logic and feeling. For example, they believed that “all men

* From *Poems of Science*, John Heath-Stubbs and Phillips Salman, eds. (New York: Penguin, 1984), pp. 150–152. We thank E.B. Sparberg for bringing this to our attention.

are created equal.” Yet they did little to remove the chains of black slaves, the ghetto walls imprisoning Jews, or the laws that denied rights to women.

Still, compared with the previous century, the dominant theme of the eighteenth century was *moderation*, the “happy medium.” The emphasis was on greater toleration of different opinions, restraint of excess, and balance of opposing forces. Even reason was not allowed to question religious faith too strongly. Atheism, which some philosophers thought would logically result from unlimited rationality, was still regarded with horror by most Europeans.

The Constitution of the United States of America is one of the most enduring achievements of this period. Its system of “checks and balances” was designed specifically to prevent any one group from getting too much power. It attempted to establish in politics a state of equilibrium of opposing trends. This equilibrium, some thought, resembled the balance between the Sun’s gravitational pull and the tendency of a planet to fly off in a straight line. If the gravitational attraction upon the planet increased without a corresponding increase in planetary speed, the planet would fall into the Sun. If the planet’s speed increased without a corresponding increase in gravitational attraction, it would escape from the solar system. When the opposing tendencies balanced, harmony resulted.

Political philosophers, some of whom used Newtonian physics as a model, hoped to create a similar balance in government. They tried to devise a system that would avoid the extremes of dictatorship and anarchy. According to James Wilson (1742–1798), who played a major role in writing the American Constitution:

In government, the perfection of the whole depends on the balance of the parts, and the balance of the parts consists in the independent exercise of their separate powers, and, when their powers are separately exercised, then in their mutual influence and operation on one another. Each part acts and is acted upon, supports and is supported, regulates and is regulated by the rest.

Both Newton’s life and his writings seemed to support the idea of political democracy. A former farm boy had attained the outermost reaches of the human imagination. What he had found there meant, first of all, that the same set of laws governed motion in the celestial and terrestrial spheres. This smashed the old beliefs about “natural place” and extended a new democracy throughout the Universe. Newton had shown that all matter, whether the Sun or an ordinary stone, was created equal; that is to say, all matter had the same standing before “the Laws of Nature and of Nature’s God.” (This phrase was used at the beginning of the Declaration of Inde-

pendence to justify the desire of the people in the American colonies to throw off their oppressive political system and to become an independent people.) All political thought at this time was heavily influenced by Newtonian ideas. The *Principia* seemed to offer a parallel to theories about democracy. It seems logical that all people, like all natural objects, are created equal before nature's creator.

In literature, too, as already indicated, many welcomed the new scientific viewpoint. It supplied new ideas, convenient figures of speech, metaphors, parallels, and concepts which writers used in poems and essays. Many poems of the eighteenth century referred to Newton's discovery that white light is composed of colors (see Chapter 8). Samuel Johnson advocated that words drawn from the vocabulary of the natural sciences be used in literary works. He defined many such words in his *Dictionary* and illustrated their application in his *Rambler* essays.

However, not everyone welcomed the new rational, scientific viewpoint. That viewpoint was based on the idea that nature consists only of matter moving through empty space according to gravity and Newton's laws of motion. Many writers and artists of the Romantic movement were particularly disturbed by this so-called "mechanical world view" which, they argued, replaced the vibrancy and beauty of nature with an ugly, lifeless world of inert particles moving forever in empty space. Where in this system is there room for the beauty and warmth and feeling of a gorgeous rainbow, a melodious concerto, or the emotions of love and hate, ambition and pride, happiness and sorrow?

Romanticism started in Germany about 1780 among young writers inspired by the poet-philosopher Johann Wolfgang von Goethe. The most familiar examples of Romanticism in English literature are the poems and novels of Blake, Coleridge, Shelley, Byron, Scott, and Wordsworth. Most of the Romantics scorned the mathematical view of nature. They believed that any whole thing, whether a single human being or the entire Universe, is filled with a unique, nonmaterial spirit. This spirit cannot be explained by reason; it can only be *felt*. The Romantics insisted that phenomena cannot be meaningfully analyzed and reduced to their separate parts by mechanical explanations or pure reason alone. Contrast the following excerpt from William Wordsworth's (1770–1850) "The Tables Turned"* with Akenside's "Hymn to Science" quoted earlier:*

Up! Up! my friend, and clear your looks,
Why all this toil and trouble?

* *ibid.*, p. 166.

Up! Up! my friend, and quit your books,
Or surely you'll grow double. . . .

Books! 'tis a dull and endless strife,
Come, hear the woodland linnet,
How sweet his music; on my life
There's more of wisdom in it.

And hark! How blithe the throstle sings!
And he is no mean preacher;
Come forth into the light of things,
Let Nature be your teacher.

The Romantic philosophers in Germany regarded Goethe as their greatest scientist as well as their greatest poet. They pointed in particular to his theory of color, which flatly contradicted Newton's theory of light. Goethe held that white light does not consist of a mixture of colors and that it is useless to "reduce" or "torture" a beam of white light by passing it through a prism to study its separate spectral colors. Rather, he charged, the colors of the spectrum are artificially produced in Newton's experiment using the prism, acting on and changing the light which is itself pure.

In the judgment of all modern scientists on this point, Newton was right and Goethe wrong. This does not mean that so-called *Nature Philosophy*, introduced by Friedrich Schelling in the early 1800s as the Romantic answer to Newtonian physics, was without any value. It encouraged speculation about ideas, even if they were so general that they could not be easily tested by experiment. At the time, it was condemned by most scientists for just this reason. Today, most historians of science agree that Nature Philosophy eventually played an important role in making possible certain scientific discoveries later on. Among these was the general principle of conservation of energy, which is described in the next two chapters. This principle asserted that all the "forces of nature," that is, the phenomena of heat, gravity, electricity, magnetism, and so forth, are forms of one underlying "force" (which we now call energy). This idea had agreed well with the viewpoint of Nature Philosophy. But it also could be put eventually in a scientifically acceptable form.

Movements hostile to conventional science have in fact occurred from time to time since Antiquity, in various forms, and are again visible today. Some modern artists, some intellectuals, and most members of the "alternative" or "new age" movements express deep-felt dislikes and mistrust of science. Their feelings are similar to, and historically related with, those of the Romantics. They are based in part on the mistaken notion that modern scientists dogmatically claim to be able to find (or have) a mechanical

explanation for *everything*, whereas science is so powerful by being neither dogmatic, nor beholden only to “mechanics,” nor ambitious to other fields in which it does not belong.

Even the Roman philosopher Lucretius (100–55 B.C.), who supported the atomic theory in his poem *On the Nature of Things*, wished to preserve some role for “free will” in the Universe, by suggesting that atoms might swerve randomly in their paths. This was not enough for Romantics, or even for some scientists. For example, Erasmus Darwin, a scientist and grandfather of evolutionist Charles Darwin, asked:

Dull atheist, could a giddy dance
Of atoms lawless hurl'd
Construct so wonderful, so wise,
So harmonised a world?

The Romantic Nature philosophers thought they could discredit the Newtonian scientists by forcing them to answer this question. To say “yes,” they argued, would be absurd, and to say “no” would be disloyal to Newtonian beliefs. But the Newtonians succeeded quite well without committing themselves to any definite answer to Erasmus Darwin’s question. They went on to discover immensely powerful and valuable laws of nature, which are discussed in the chapters ahead.

Questions

1. Describe some of the impacts of Newton’s work outside the field of science.
2. What impact did Newtonian physics have on political thought?
3. Why did some people eventually reject the new physics?
4. Contrast the excerpts from the poems by Akenside and Wordsworth.
5. The poem by Erasmus Darwin asks a question. What is it in your own words? How did Nature Philosophers attempt to discredit Newtonian scientists?

CHAPTER 5. CONSERVING MATTER AND MOTION

An Example of Conservation of Momentum

(1) A space capsule at rest in space, far from the Sun or planets, has a mass of 1000 kg. A meteorite with a mass of 0.1 kg moves toward it with a speed

of 1000 m/s. How fast does the capsule (with the meteorite stuck in it) move after being hit?

$$m_A \text{ (mass of the meteorite)} = 0.1 \text{ kg,}$$

$$m_B \text{ (mass of the capsule)} = 1000 \text{ kg,}$$

$$v_A \text{ (initial speed of meteorite)} = 1000 \text{ m/s,}$$

$$v_B \text{ (initial speed of capsule)} = 0,$$

$$v'_A \text{ (final speed of meteorite)} = ?,$$

$$v'_B \text{ (final speed of capsule)} = ?.$$

The law of conservation of momentum states

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}'_A + m_B \mathbf{v}'_B.$$

Inserting the values given, we have

$$(0.1 \text{ kg})(1000 \text{ m/s}) + (1000 \text{ kg})(0)$$

$$= (0.1 \text{ kg}) \mathbf{v}'_A + (1000 \text{ kg}) \mathbf{v}'_B,$$

$$100 \text{ kg} \cdot \text{m/s} = (0.1 \text{ kg}) \mathbf{v}'_A + (1000 \text{ kg}) \mathbf{v}'_B.$$

Since the meteorite sticks to the capsule, $\mathbf{v}'_B = \mathbf{v}'_A$; so we can write

$$100 \text{ kg} \cdot \text{m/s} = (0.1 \text{ kg}) v'_A + (1000 \text{ kg}) v'_A,$$

$$100 \text{ kg} \cdot \text{m/s} = (1000 \cdot 1 \text{ kg}) v'_A.$$

Therefore,

$$v'_A = \frac{100 \text{ kg} \cdot \text{m/s}}{1000.1 \text{ kg}}$$

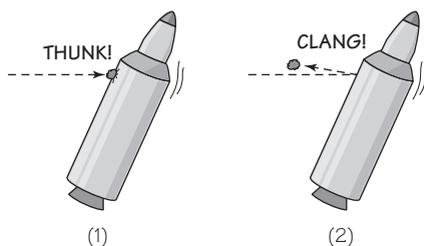
$$= 0.1 \text{ m/s}$$

(in the original direction of the motion of the meteorite). Thus, the capsule (with the stuck meteorite) moves on with a speed of 0.1 m/s.

Another approach to the solution is to handle the symbols first, and substitute the values as a final step. Substituting \mathbf{v}'_A for \mathbf{v}'_B and letting $\mathbf{v}'_B = 0$ would leave the equation $m_A\mathbf{v}_A = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B = (m_A + m_B)\mathbf{v}$. Solving for \mathbf{v}'_A we obtain

$$\mathbf{v}'_A = \frac{m_A\mathbf{v}_A}{(m_A + m_B)}.$$

This equation holds true for any projectile hitting (and staying with) a body initially at rest that moves on in a straight line after collision.



(2) An identical capsule at rest nearby is hit by a meteorite of the same mass as the other. However, this meteorite, hitting another part of the capsule, does not penetrate. Instead, it bounces straight back with almost no change of speed. How fast does the capsule move after being hit? Since all these motions are assumed to be along a straight line, we can drop the vector notation from the symbols and indicate the reversal in direction of the meteorite with a minus sign.

The same symbols are appropriate as in (1):

$$m_A = 0.1 \text{ kg}, \quad v_B = 0,$$

$$m_B = 1000 \text{ kg}, \quad v'_A = 1000 \text{ m/s},$$

$$v_A = 1000 \text{ m/s}, \quad v'_B = ?.$$

The law of conservation of momentum states

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B.$$

Here,

$$\begin{aligned} & (0.1 \text{ kg})(1000 \text{ m/s}) + (1000 \text{ kg})(0), \\ & = (0.1 \text{ kg})(-1000 \text{ m/s}) + (1000 \text{ kg}) v'_B \\ & 100 \text{ kg} \cdot \text{m/s} = -100 \text{ kg} \cdot \text{m/s} + (1000 \text{ kg}) v'_B, \\ & v'_B = \frac{200 \text{ kg} \cdot \text{m/s}}{1000 \text{ kg}} = 0.2 \text{ m/s}. \end{aligned}$$

Thus, the struck capsule moves on with about twice the speed of the capsule in (1). (A general symbolic approach to this solution can be taken, too. The result is valid only for the special case of a projectile rebounding perfectly elastically from a body of much greater mass.)

There is a general lesson here. It follows from the law of conservation of momentum that a struck object is given less momentum if it absorbs the projectile than if it reflects it. (A goalie who catches the soccer ball is pushed back less than one who lets the ball bounce off.) Some thought will help you to understand this idea: An interaction that merely stops the projectile is not as great as an interaction that first stops it and then propels it back again.

Doing Work on a Sled

Suppose a loaded sled of mass m is initially at rest on low-friction ice. You, wearing spiked shoes, exert a constant horizontal force F on the sled. The weight of the sled is balanced by the upward push exerted by the ice, so F is effectively the net force on the sled. You keep pushing, running faster and faster as the sled accelerates, until the sled has moved a total distance d .

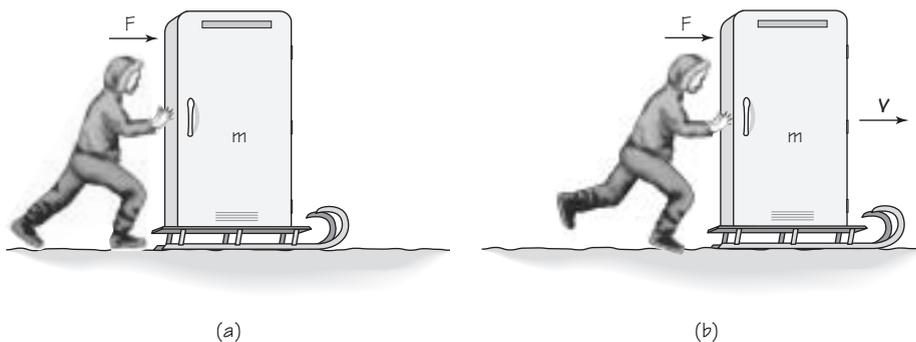
If the net force F is constant, the acceleration of the sled is constant. Two equations that apply to motion starting from rest with constant acceleration are

$$v = at$$

and

$$d = \frac{1}{2}at^2,$$

where a is the acceleration of the body, t is the time interval during which it accelerates (i.e., the time interval during which a net force acts on the body), v is the final speed of the body, and d is the distance it moves in the time interval t .



According to the first equation, $t = v/a$. If we substitute this expression for t in the second equation, we obtain

$$d = \frac{1}{2}at^2 = \frac{1}{2}a\frac{v^2}{a^2} = \frac{1}{2}\frac{v^2}{a}.$$

The work done on the sled is $W = Fd$. From Newton's second law, $F = ma$, so

$$\begin{aligned} W &= Fd \\ &= ma \times \frac{1}{2}\frac{v^2}{a^2}. \end{aligned}$$

The acceleration cancels out, giving

$$W = \frac{1}{2}mv^2.$$

Therefore, the work done in this case can be found from just the mass of the body and its final speed. With more advanced mathematics, it can be shown that the result is the same whether the force is constant or not.

More generally, we can show that the change in kinetic energy of a body already moving is equal to the work done on the body. By the definition of average speed

$$d = v_{\text{av}}t.$$

If we consider a uniformly accelerated body whose speed changes from v_0 to v , the average speed (v_{av}) during t is $\frac{1}{2}(v + v_0)$. Thus,

$$d = \frac{v + v_0}{2} \times t.$$

By the definition of acceleration, $a = \Delta v/t$; therefore, $t = \Delta v/a = (v - v_0)/a$. Substituting $(v - v_0)/a$ for t gives

$$\begin{aligned} d &= \frac{v + v_0}{2} \times \frac{v - v_0}{a} \\ &= \frac{(v + v_0)(v - v_0)}{2a} \\ &= \frac{v^2 - v_0^2}{2a}. \end{aligned}$$

The work W done is $W = Fd$, or, since $F = ma$:

$$\begin{aligned} W &= ma \times d \\ &= ma \times \frac{v^2 - v_0^2}{2a} \\ &= \frac{m}{2} (v^2 - v_0^2) \\ &= \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2. \end{aligned}$$

CHAPTER 6. THE DYNAMICS OF HEAT

You will often see energies expressed in terms of other units. A few of them are listed here.

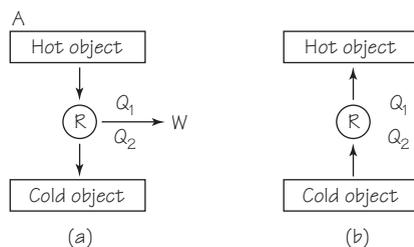
<i>Unit name</i>	<i>Symbol</i>	<i>Definition</i>	<i>Conversion</i>
kilowatt hour	kWhr	A watt (W) is 1 J/s, so 1 J = 1 W · s. A kWh is the amount of energy delivered in 1 hr if 1 kJ is delivered per second.	1 kWh = 3.60 MJ
Calorie (or kilocalorie)	Cal (or kcal)	The energy required to heat 1 kg of water by 1°C.	4.19 kJ
British thermal unit	Btu	The energy required to heat 0.454 kg by 0.556°C.	1.06 kJ

Carnot's Proof

Carnot's proof of maximum efficiency of ideal, reversible engines starts with the premise that when a cold object is in contact with a warmer one, the cold object does not spontaneously cool itself further and so give more heat to the warm object. However, an engine placed between the two bodies *can* move heat from a cold object to a hot one. Thus, a refrigerator can cool a cold bottle further, ejecting heat into the hot room. *You will see that this is not simple.* Carnot proposed that during any such experiment, the net result cannot be *only* the transfer of a given quantity of heat from a cold body to a hot one.

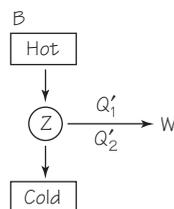
The engines considered in this case all work in cycles. At the end of each cycle, the engine itself is back to where it started. During each cycle, it has taken up and given off heat, and it has exerted forces and done work.

Consider an engine, labeled R in the figure, which suffers no internal friction, loses no heat because of poor insulation, and runs so perfectly that it can work backward in exactly the same way as forward (Figure A).

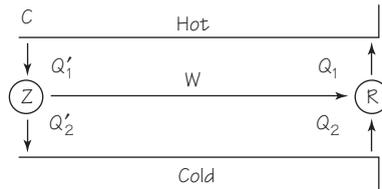


Now suppose someone claims to have invented an engine, labeled Z in the next figure, which is even more efficient than the ideal engine R. That is, in one cycle it makes available the same amount of work, W , as the R engine does, but takes less heat energy, Q' , from the hot object to do it ($Q'_1 < Q_1$). Since heat and energy are equivalent and since $Q_2 = Q_1 - W$ and $Q'_2 = Q'_1 - W$, it will also be true that $Q'_2 < Q_2$ (Figure B).

$$Q'_2 = Q'_1 - W,$$



Suppose the two engines are connected so that the work from one can be used to drive the other. For example, the Z engine can be used to make the R engine work like a refrigerator (Figure C).



At the end of one cycle, both Z and R are back where they started. No work has been done; the Z engine has transferred some heat to the cold object; and the R engine has transferred some heat to the hot object. The *net* heat transferred is $Q_1 - Q'_1$, and the net heat taken from the cold object is $Q_2 - Q'_2$. These are, in fact, the same

$$\begin{aligned} Q_2 - Q'_2 &= (Q_1 - W) - (Q'_1 - W) \\ &= Q_1 - Q'_1. \end{aligned}$$

Because Z is supposed to be more efficient than R, this quantity should be positive; that is, heat has been transferred from the cold object to the hot object. Nothing else has happened. But, according to the fundamental premise, this is impossible, and does not happen.

The only conclusion is that the Z engine was improperly “advertised” and that it is either impossible to build or that in actual operation it will turn out to be *less* efficient than R.

As for two different reversible engines, they must have the same efficiency. Suppose the efficiencies were different; then one would have to be more efficient than the other. What happens when the more efficient engine is used to drive the other reversible engine as a refrigerator? The same argument just used shows that heat would be transferred from a cold body to a hot one. This is impossible. Therefore, the two reversible engines must have the same efficiency.

To actually compute that efficiency, you must know the properties of one reversible engine; all reversible engines working between the same temperatures must have that same efficiency. (Carnot computed the efficiency of an engine that used an ideal gas instead of steam.)

CHAPTER 7. HEAT—A MATTER OF MOTION

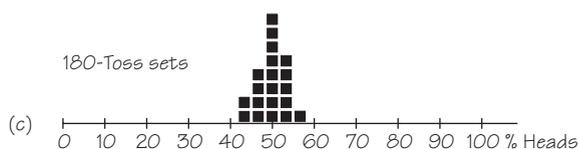
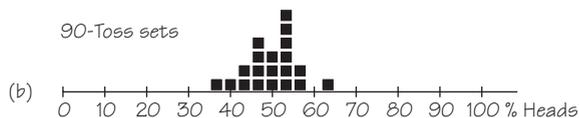
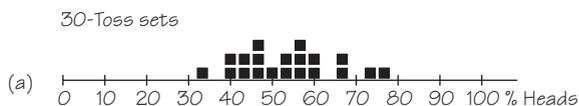
Averages and Fluctuations

Molecules are too small, too numerous, and too fast for us to measure the speed of any one molecule, its kinetic energy, or how far it moves before

colliding with another molecule. For this reason, the kinetic theory of gases concerns itself with making predictions about *average* values. The theory enables us to predict quite precisely the *average* speed of the molecules in a sample of gas, the *average* kinetic energy, or the *average* distance the molecules move between collisions.

Any measurement made on a sample of gas reflects the combined effect of billions of molecules, averaged over some interval of time. Such average values measured at different times, or in different parts of the sample, will be slightly different. We assume that the molecules are moving randomly. Thus, we can use the mathematical rules of statistics to estimate just how different the averages are likely to be. We will call on two basic rules of statistics for random samples:

1. Large variations away from the average are less likely to occur than are small variations. (For example, if you toss 10 coins, you are less likely to get 9 heads and 1 tail than to get 6 heads and 4 tails.)
2. Percentage variations are likely to be smaller for large samples. (For example, you are likely to get nearer to 50% heads by flipping 1000 coins than by flipping just 10 coins.)



A simple statistical prediction is the statement that if a coin is tossed many times, it will land “heads” 50% of the time and “tails” 50% of the time. For small sets of tosses there will be many “fluctuations” (variations) to either side of the predicted average of 50% heads. Both statistical rules are evident in the charts. The top chart shows the percentage of heads in sets of 30 tosses each. Each of the 10 black squares represents a set of 30 tosses. Its position along the horizontal scale indicates the percentage of

heads. As we would expect from Rule 1, there are more values near the theoretical 50% than far from it. The second chart is similar to the first, but here each square represents a set of 90 tosses. As before, there are more values near 50% than far from it. And, as we would expect from Rule 2, there are fewer values far from 50% than in the first chart.

The third chart is similar to the first two, but now each square represents a set of 180 tosses. Large fluctuations from 50% are less common still than for the smaller sets.

Statistical theory shows that the *average* fluctuation from 50% shrinks in proportion to the square root of the number of tosses. We can use this rule to compare the average fluctuation for sets of, say, 30,000,000 tosses with the average fluctuation for sets of 30 tosses. The 30,000,000-toss sets have 1,000,000 times as many tosses as the 30-toss sets. Thus, their average fluctuation in percent of “heads” should be 1,000 times smaller!

These same principles hold for fluctuations from average values of any randomly distributed quantities, such as molecular speed or distance between collisions. Since even a small bubble of air contains about a quintillion (10^{18}) molecules, fluctuations in the average value for any isolated sample of gas are not likely to be large enough to be measurable. A measurably large fluctuation is not *impossible*, but extremely unlikely.

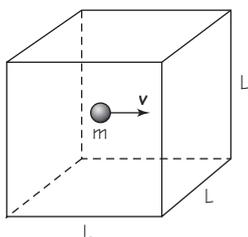
Deriving an Expression for Pressure from the Kinetic Theory

We begin with the model of a gas described in Section 7.2: “a large number of very small particles in rapid, disordered motion.” We can assume here that the particles are points with vanishingly small size, so that collisions between them can be ignored. If the particles did have finite size, the results of the calculation would be slightly different. But the approximation used here is accurate enough for most purposes.

The motions of particles moving in all directions with many different velocities are too complex as a starting point for a model. So we fix our attention first on one particle that is simply bouncing back and forth between two opposite walls of a box. Hardly any molecules in a real gas would actually move like this. But we will begin here in this simple way and later in this chapter extend the argument to include other motions. This later part of the argument will require that one of the walls be movable. Therefore, we will arrange for that wall to be movable, but to fit snugly into the box.

In Chapter 5, you saw how the laws of conservation of momentum and energy apply to cases like this. When a very light particle hits a more massive object, like the wall, very little kinetic energy is transferred. If the collision is elastic, the particle will reverse its direction with very little change

in speed. In fact, if a force on the outside of the wall keeps it stationary against the impact from inside, the wall will not move during the collisions. Thus *no work* is done on it, and the particles rebound without any change in speed.



How large a force will these particles exert on the wall when they hit it? By Newton's third law the average force acting on the wall is equal and opposite to the average force with which the wall acts on the particles. The force on each particle is equal to the product of its mass times its acceleration ($\mathbf{F} = m\mathbf{a}$), by Newton's second law. The force can also be written as

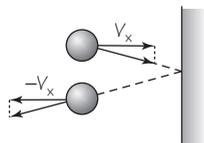
$$\mathbf{F} = \frac{\Delta(m\mathbf{v})}{\Delta t},$$

where $\Delta m\mathbf{v}$ is the change in momentum. Thus, to find the average force acting on the wall we need to find the change in momentum per second due to molecule-wall collisions.

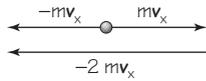
Imagine that a particle, moving with speed v_x (the component of \mathbf{v} in the x direction) is about to collide with the wall at the right. The component of the particle's momentum in the x direction is mv_x . Since the particle collides elastically with the wall, it rebounds with the same speed. Therefore, the momentum in the x direction after the collision is $m(-v_x)$. The change in the momentum of the particle as a result of this collision is

final momentum – initial momentum = change in momentum,

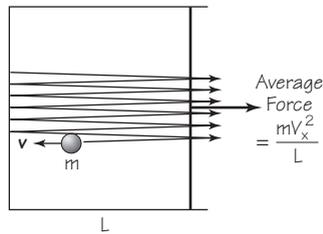
$$(-mv_x) - (mv_x) = (-2mv_x).$$



Note that all the vector quantities considered in this derivation have only two possible directions: to the right or to the left. We can therefore indicate direction by using a + or a - sign, respectively.



Now think of a single particle of mass m moving in a cubical container of volume L^3 as shown in the figure.



The time between collisions of one particle with the right-hand wall is the time required to cover a distance $2L$ at a speed of v_x ; that is, $2L/v_x$. If $2L/v_x$ equals the time between collisions, then $v_x/2L$ equals the number of collisions per second. Thus, the change in momentum per second is given by

$$\left(\begin{array}{c} \text{change in momentum} \\ \text{in one collision} \end{array} \right) \times \left(\begin{array}{c} \text{number of collisions} \\ \text{per second} \end{array} \right) = \left(\begin{array}{c} \text{change in momentum} \\ \text{per second} \end{array} \right),$$

$$(-2mv_x) \times \frac{v_x}{2L} = \frac{-mv_x^2}{L}.$$

The net force equals the rate of change of momentum. Thus, the average force acting on the molecule (due to the wall) is equal to $-mv_x^2/L$, and by Newton's third law, the average force acting on the wall (due to the molecule) is equal to $+mv_x^2/L$. So the average pressure on the wall due to the collisions made by one molecule moving with speed v_x is

$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V},$$

where V (here L^3) is the volume of the cubical container.

Actually, there are not one but N molecules in the container. They do not all have the same speed, but we need only the average speed in order

to find the pressure they exert. More precisely, we need the average of the square of their speeds in the x direction. We call this quantity $(v_x^2)_{\text{av}}$. The pressure on the wall due to N molecules will be N times the pressure due to one molecule, or

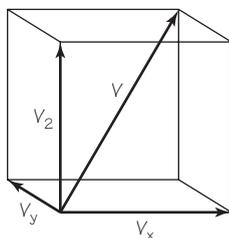
$$P = \frac{Nm(v_x^2)_{\text{av}}}{V}.$$

In a real gas, the molecules will be moving in all directions, not just in the x direction; that is, a molecule moving with speed v will have three components: v_x , v_y , and v_z . If the motion is random, then there is no preferred direction of motion for a large collection of molecules, and $(v_x^2)_{\text{av}} = (v_y^2)_{\text{av}} = (v_z^2)_{\text{av}}$. It can be shown from Pythagoras' theorem that $v^2 = v_x^2 + v_y^2 + v_z^2$. These last two expressions can be combined to give

$$(v^2)_{\text{av}} = 3(v_x^2)_{\text{av}}$$

or

$$(v_x^2)_{\text{av}} = 1/3(v^2)_{\text{av}}.$$



By substituting this expression for $(v_x^2)_{\text{av}}$ in the pressure formula, we get

$$\begin{aligned} P &= \frac{Nm \times 1/3(v^2)_{\text{av}}}{V} \\ &= 1/3 \frac{Nm}{V} (v^2)_{\text{av}}. \end{aligned}$$

Notice now that Nm is the total mass of the gas, and therefore Nm/V is just the density D . So

$$P = 1/3 D (v^2)_{\text{av}}.$$

This is our theoretical expression for the pressure P exerted on a wall by a gas in terms of its density D and the molecular speed v .

Now if d is very small compared to l , as you can easily arrange in practice, the circular arc S_2M will be a very small piece of a large-diameter circle, or nearly a straight line. Also, the angle S_1MS_2 is very nearly 90° . Thus, the triangle S_1S_2M can be regarded as a right triangle. Furthermore, angle S_1S_2M is equal to angle POQ . Then the right triangle S_1S_2M is similar to triangle POQ :

$$\frac{S_1M}{S_1S_2} = \frac{X}{OP} \quad \text{or} \quad \frac{S_1M}{d} = \frac{X}{L}.$$

If the distance l is large compared to x , the distances l and L are nearly equal. Therefore,

$$\frac{S_1M}{d} = \frac{x}{l}$$

But S_1M is the extra distance traveled by the wave from source S_1 . For P to be a point of maximum wave disturbance, S_1M must be equal to $n\lambda$ (where $n = 0$ if P is at Q , and $n = 1$ if P is at the first maximum of wave disturbance found to one side of Q , etc.). So the equation becomes

$$\frac{n\lambda}{d} = \frac{x}{l}$$

and

$$\lambda = \frac{dx}{nl}.$$

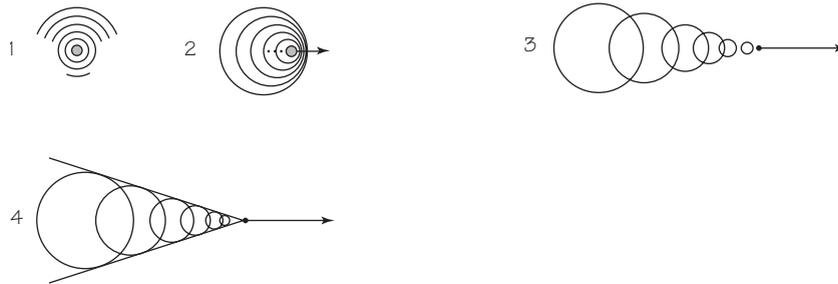
This important result says that if you measure the source separation d , the distance l , and the distance x from the central line to a wave disturbance maximum, you can calculate the wavelength λ .

The Sonic Boom

In the last half century a new kind of noise has appeared: the sonic boom. An explosion-like sonic boom is produced whenever an object travels through air at a speed greater than the speed of sound (supersonic speed). Sound travels in air at about 340 m/s. Many types of military airplanes can travel at two or three times this speed. Flying at such speeds, the planes

unavoidably and continually produce sonic booms, which can cause physical damage, and anxiety in people and animals. SST (Supersonic Transport) planes such as the *Concorde* are now in civilian use in some countries. The unavoidable boom raises important questions. What are the consequences of this technological “progress”? Who gains, and what fraction of the population do they represent? Who and how many pay the price? *Must* we pay it; must SST’s be used? How much say has the citizen in decisions that affect the environment so violently?

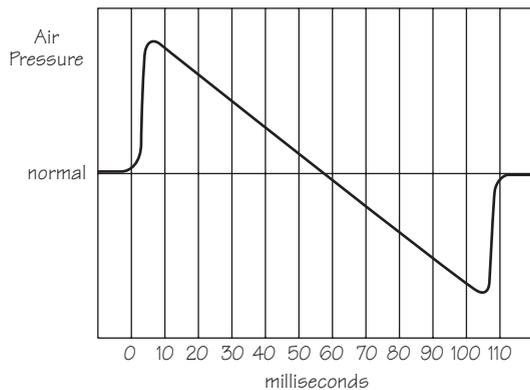
The formation of a sonic boom is similar to the formation of a wake by a boat. Consider a simple point source of waves. If the source remains in the same position in a medium, the wave it produces spreads out symmetrically around it, as in Diagram 1. If the source of the disturbance is *moving* through the medium, each new crest starts from a different point, as in Diagram 2.



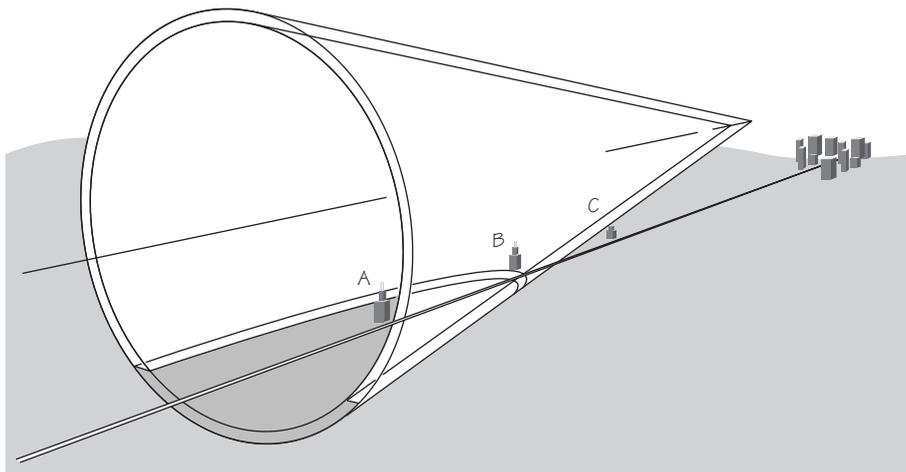
Notice that the wavelength has become shorter in front of the object and longer behind it. This is called the *Doppler effect*. The Doppler effect is the reason that the sound an object makes seems to have a higher pitch when it is moving toward you and a lower pitch when it is moving away from you. In Diagram 3, the source is moving through the medium *faster than the wave speed*. Thus, the crests and the corresponding troughs overlap and interfere with one another. The interference is mostly destructive everywhere except on the line tangent to the wave fronts, indicated in Diagram 4. The result is a wake that spreads like a wedge away from the moving source, as in the diagram.

All these concepts apply not only to water waves but also to sound waves, including those disturbances set up in air by a moving plane as it pushes the air out of the way. If the source of sound is moving faster than the speed of sound wave, then there is a cone-shaped wake (in three dimensions) that spreads away from the source.

Actually, two cones of sharp pressure change are formed. One cone originates at the front of the airplane and one at the rear, as indicated in the graph at the right.



Because the double shock wave follows along behind the airplane, the region on the ground where people and houses may be struck by the boom (the “sonic-boom carpet”) is as long as the supersonic flight path itself. In such an area, typically thousands of kilometers long and 80 km wide, there may be millions of people. Tests made with airplanes flying at supersonic speed have shown that a single such cross-country flight by a 315-ton supersonic transport plane would break many thousands of dollars worth of windows, plaster walls, etc., and cause fright and annoyance to millions of people. Thus, the supersonic flight of such planes has been confined to over-ocean use. It may even turn out that the annoyance to people on ship-board, on islands, and on coastal areas near the flight paths is so great that over-ocean flights, too, will have to be restricted.



Model, Analogy, Hypothesis, Theory

Model, analogy, hypothesis, and *theory* have similar but distinct meanings when applied to physics. An *analogy* is a corresponding situation which, though perhaps totally unrelated to the situation at hand, helps you understand it. Many electronic circuits have analogs in mechanical systems. A *model* is a corresponding situation that may offer a picture of what “is really going on” and therefore can be taken more seriously as an explanation. An electron rotating around a nucleus is one model for an atom. A *hypothesis* is a statement that can usually be directly or indirectly tested. To Franklin, the statement “lightning is caused by electricity” was at first a hypothesis. A *theory* is a more general construction, perhaps putting together several models and hypotheses to explain a collection of effects that previously seemed unrelated. Newton’s explanation of Kepler’s laws, Galileo’s experiments in mechanics and, finally, the Cavendish experiment were all part of the theory of universal gravitation. This is a good example of a theory.

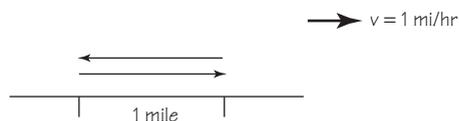
A well-tested theory, such as Newton’s theory of gravitation or Einstein’s theory of relativity, is a robust part of science, explaining a myriad of individual events or facts, and not to be confused with the vernacular use of “just a theory.”

CHAPTER 9. EINSTEIN AND RELATIVITY THEORY

Differences in Speed for Light Waves Traveling Parallel and Perpendicular to the Ether Wind

Instead of light waves moving parallel and perpendicular to the ether wind, we examine an equivalent situation: a swimmer swimming at constant speed, first parallel and perpendicular to a current, in a river 1 mi wide. Assume the swimmer can swim at 2 mi/hr and the stream runs 1 mi/hr from left to right in the diagram below. We will calculate the time required for the swimmer to travel 1 mi each way with and against the current and 1 mi back and forth across the current.

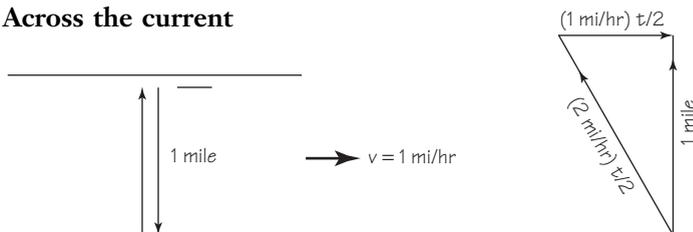
With and against the current



Traveling 1 mi with the current, the swimmer's speed is enhanced by the speed of the current, while traveling 1 mi back against the current, his speed is hindered by it. Thus the total time for the round trip is

$$\begin{aligned} t &= \frac{1 \text{ mi}}{2 \text{ mi/hr} + 1 \text{ mi/hr}} + \frac{1 \text{ mi}}{2 \text{ mi/hr} - 1 \text{ mi/hr}} \\ &= \frac{1 \text{ mi}}{3 \text{ mi/hr}} + \frac{1 \text{ mi}}{1 \text{ mi/hr}} \\ &= 1.33 \text{ hr} \end{aligned}$$

Across the current



In order to swim directly across the river from the starting point and back, the swimmer, in each direction, must head toward a point upstream from the destination point. The path taken *relative to the fixed shore* will be directly across and back, 1 mi in each direction. But the path taken in each direction by the swimmer *relative to the flowing water* will be along the hypotenuse of a right triangle formed by the 1-mi width of the river and the speed of the river current times one-half the total time for the round trip $(1 \text{ mi/hr})t/2$.

Using the Pythagorean theorem, the total distance traveled by the swimmer at the speed of 2 mi/hr is

$$(2 \text{ mi/hr}) t = 2\sqrt{(1 \text{ mi})^2 + [(1 \text{ mi/hr}) t/2]^2}.$$

In order to solve for t , cancel 2 on both sides and square both sides to get

$$\text{mi}^2/\text{hr}^2 t^2 = \text{mi}^2 + \frac{1}{4} \text{mi}^2/\text{hr}^2 t^2.$$

Cancel mi^2 , multiply through by hr^2 and solve for t^2 :

$$\frac{3}{4}t^2 = 1 \text{ hr}^2,$$

$$t = \sqrt{4/3} \text{ hr} = 1.15 \text{ hr}.$$

Note that the time to cross the river back and forth at constant swimming is less than the time it took in the earlier example to swim the same distance parallel to the current and back.

Michelson and Morley reasoned that exactly the same kind of result would occur for a light beam split in half—one-half sent “swimming” perpendicular to the supposed ether wind and back, the other half “swimming” parallel to the wind and back. Although the two halves of the beam started out together, the one sent parallel to the wind should return slightly behind the one sent perpendicular to the wind. The difference in time was expected to be small but detectable. Yet, when comparing the two light waves experimentally, they could find *no difference in the time of travel* of the two beams. We now know from Einstein’s second postulate that the times had to be the same, and that the ether model, while usually appealing, is misleading.

