Some Further Chapter Materials

PART TWO

CHAPTER 10. ELECTRICITY AND MAGNETISM

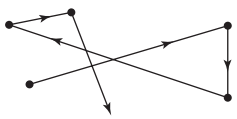
Electrical Conduction in Metals

While charges cannot move freely through an insulator, they can move freely through a conductor. Yet when a conductor (say, a piece of copper wire or a steel knife blade) is connected between the two terminals of a battery, a steady current starts immediately and persists until the battery is discharged. This is puzzling. The battery sets up a potential difference between the two ends of the conductor and so there is an electric field along the conductor. This means that there is an electrical force on the charges. If this were the net force on the charges, they would be moving faster and faster. In that case, the current should increase with time, a situation not at all like what actually happens.

An acceptable model for a conductor must be a little more complex, then, than a substance "through which charge can move freely." One of the first useful models for a conductor (and one which is still used today) was con-

structed around 1900 by Drude and Lorentz. They pictured the atoms of a perfect crystal of metal locked into position in a regular array (called a lattice). Each atom has one or more electrons (depending on the metal) that are shared with all the other atoms in the metal. These mobile electrons are always in random motion at very high speeds (roughly 10^6 m/s for copper), very much like the molecules of a gas studied in Chapter 7. The electrons' motion is *much* faster, though, than that of the gas molecules at the same temperature (the reason for this was not discovered until about 1930 when quantum mechanics was applied to the problem).

An electric current exists where there is *net* flow of charge along the wire. As long as the electrons are moving at random, the net flow is zero, on average. The electrons are constantly experiencing collisions with any metal atom which gets "out of line," for example, impurities in the metal or imperfections in the lattice, and with vibrations of the atoms caused by their own random thermal motion. On average, an electron travels freely for a time t between consecutive collisions (for copper, this time t is about 10^{-14} s).

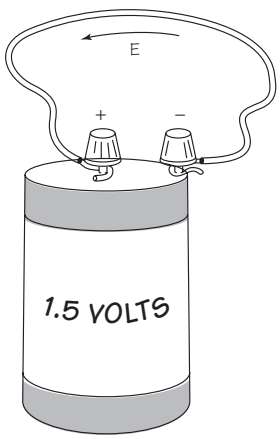


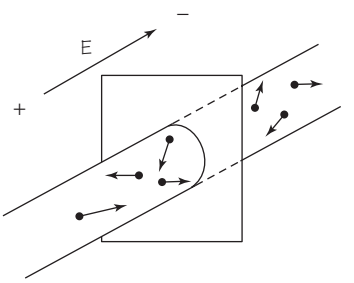
Path of an electron.

When a battery is connected to the metal, there is an electric field E created along the length of the conductor. This field does indeed accelerate the electrons, but since they move freely only for a time t, the change in their velocity caused by the field is just

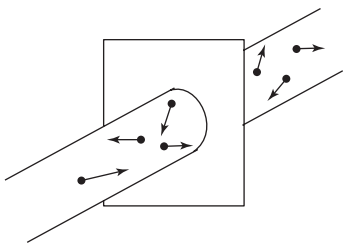
$$\Delta v = at$$

$$= E\frac{q_e}{m}t.$$





No net flow of electrons past the surface.



A net flow of electrons past the surface.

This *additional* velocity imparted to the electrons is called the "drift velocity" and is responsible for the conduction of electricity. Since E is proportional to the battery's voltage, it is easy to see that the current will be proportional to the voltage (Ohm's law) so long as the average time between collisions, t, does not change. For example, when a metal is cooled, the thermal motion of the atoms is reduced and collisions with these thermal vibrations become less frequent. Therefore, cooling a metal makes it a better conductor. Similarly, a very pure sample of copper is a better conductor than a sample with many impurities from which electrons are scattered as they move. A more quantitative model can also be described (though that is not necessary in understanding the basic model). Picture a piece of wire of length L, cross-sectional area A, with an average of n electrons in each cubic centimeter.

Ignore the *random* motion of the electrons, since this makes no contribution to the conduction, and picture all the electrons moving with the drift velocity

$$v_d = \Delta v = E\frac{q_e}{m}t.$$

The current is just the amount of charge crossing the surface each second:

$$I = \begin{pmatrix} \text{number of electrons} \\ \text{crossing surface in 1 s} \end{pmatrix} \times q_e.$$

The number of electrons crossing the surface each second is nAv_d (just as you calculated in Chapter 7 for gas molecules). Thus,

$$I = \left(\frac{nq_e^2 t_A}{m}\right)E.$$

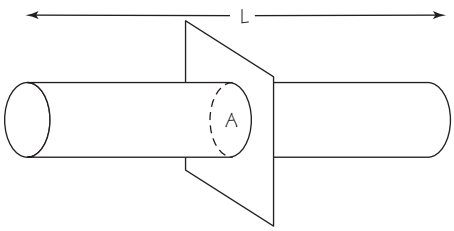
But $E = V/L$ if the wire is uniform so that the field is a constant along its length, and

$$I = \left(\frac{nq_e^2 t_A}{m}\,\frac{A}{L}\right)V.$$

That is, $I \propto V$. But this is Ohm's law! Thus, this model determines the resistance of a wire as

$$R = \left(\frac{mL}{nq_e^2 t_A}\right),$$

where $R = V/I$. It follows that, for a given material, doubling the length should double the resistance; doubling the cross-sectional area should halve the resistance. This is just what is found experimentally.



CHAPTER 14. A QUANTUM MODEL OF THE ATOM

Bohr's Quantization Rule and the Size of Orbits

The magnitude of the charge on the electron is q_e; the charge on a nucleus is Zq_e, and for hydrogen ($Z = 1$) it is just q_e. The electric force with which the hydrogen nucleus attracts its electron is therefore

$$F_{\text{el}} = k\,\frac{q_e q_e}{r^2},$$

where k is the coulomb constant, and r is the center-to-center distance. If the electron is in a stable circular orbit of radius r around the nucleus, mov-

ing at a constant speed v, then the centripetal force is equal to mv^2/r. Since the centripetal force is provided by the electric attraction,

$$\frac{mv^2}{r} = k\,\frac{q_e^2}{r^2}.$$

In the last equation, m, q_e, and k are constants; r and v are variables, whose values are related by the equation. What are the possible values of v and r for stationary states of the atom?

You can begin to get an answer if you write the last equation in slightly different form. Multiplying both sides by r^2 and dividing both sides by v, you get

$$mvr = \frac{kq_e^2}{v}.$$

The quantity on the left side of this equation is the product of the momentum of the electron and the radius of the orbit. You can use this quantity to characterize the stable orbits. According to classical mechanics, the radius of the orbit could have any value, so the quantity mvr could also have any value. Of course, classical physics also seemed to deny that there could be *any* stable orbits in the hydrogen atom. But Bohr's first postulate implies that certain stable orbits (and only those) are permitted. So Bohr needed to find the rule that decides *which* stable orbits are possible. Here Bohr appears to have been largely guided by his intuition. He found that what was needed was the recognition that the quantity mvr does not take on just any value, but only certain *allowed values*. These values are defined by the relation

$$mvr = n\,\frac{h}{2\pi},$$

where h is Planck's constant, and n is a positive integer; that is, $n = 1, 2, 3, 4, \ldots$ (but not zero). When the possible values of mvr are restricted in this way, the quantity mvr is said to be *quantized*. The integer n that appears in the formula is called the *quantum number*. The main point is that each quantum number ($n = 1, 2, 3 \ldots$) corresponds to one allowed, stable orbit of the electron.

If you accept this rule, you can at once describe the "allowed" states of the atom, for example, in terms of the radii r of the possible orbits. You

can combine the last expression above with the classical centripetal force relation as follows. The quantization rule is

$$mvr = n\,\frac{h}{2\pi}$$

so

$$r = \frac{nh}{2\pi mv}$$

and

$$r^2 = \frac{n^2h^2}{4\pi^2m^2v^2}.$$

From classical mechanics

$$\frac{mv^2}{r} = k\,\frac{q_e^2}{r}$$

so

$$v^2 = \frac{kq_e^2}{mr}.$$

Substituting this "classical" value for v^2 into the quantization expression for r^2 gives

$$r^2 = \frac{n^2h^2}{4\pi^2m^2\left(\dfrac{kq_e^2}{mr}\right)}$$

Simplifying, you get the expression for the allowed radii, r_n:

$$r_n = \frac{n^2h^2}{4\pi^2kmq_e^2} = \left(\frac{h^2}{4\pi^2kmq_e^2}\right)n^2.$$

CHAPTER 15. QUANTUM MECHANICS

The de Broglie Wavelength: Examples

A body of mass 1 kg moves with a speed of 1 m/s. What is the de Broglie wavelength?

$$\lambda = \frac{h}{mv},$$

$$h = 6.6 \times 10^{-34}\ \text{J} \cdot \text{s},$$

$$mv = 1\ \text{kg} \cdot \text{m/s},$$

$$\lambda = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{s}}{1\ \text{kg} \cdot \text{m/s}},$$

so

$$\lambda = 6.6 \times 10^{-34}\ \text{m}.$$

The de Broglie wavelength is many orders of magnitude smaller than an atom. Thus, it is much too small to be detected. There are, for example, no slits or obstacles small enough to show diffraction effects. You would expect to detect no wave aspects in the motion of this body.

An electron mass 9.1×10^{-31} kg moves with a speed of 2×10^{6} m/s. What is its de Broglie wavelength?

$$\lambda = \frac{h}{mv},$$

$$h = 6.6 \times 10^{-34}\ \text{J} \cdot \text{s},$$

$$mv = 1.82 \times 10^{-24}\ \text{kg} \cdot \text{m/s},$$

$$\lambda = \frac{6.6 \times 10^{-34}\ \text{J} \cdot \text{s}}{1.82 \times 10^{-24}\ \text{kg} \cdot \text{m/s}},$$

so

$$\lambda = 3.6 \times 10^{-10}\ \text{m}.$$

The de Broglie wavelength is of atomic dimensions. For example, it is of the same order of magnitude as the distances between atoms in a crystal. So you can expect to see wave aspects in the interaction of electrons with crystals.

The Uncertainty Principle: Examples

Applied to a large mass

Consider a car, with a mass of 1000 kg, moving with a speed of about 1 m/s. Suppose that in this experiment the inherent uncertainty Δv in the measured speed is 0.1 m/s (10% of the speed). What is the minimum uncertainty in the position of the car?

$$\Delta x\, \Delta p \geq \frac{h}{2\pi},$$

$$\Delta p = m\, \Delta v = 100 \text{ kg} \cdot \text{m/s},$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s},$$

$$\Delta x \geq \frac{6.63}{6.28} \times \frac{10^{-34} \text{ J} \cdot \text{s}}{10^2 \text{ kg} \cdot \text{m/s}},$$

$$\Delta x \geq 1 \times 10^{-36} \text{ m}.$$

This uncertainty in position, which is many orders smaller than the size of an atom, is much too small to be observable. In this case, you can determine the position of the body with as high an accuracy as you would ever need.

Applied to a small mass

Consider an electron, with a mass of 9.1×10^{-31} kg, moving with a speed of about 2×10^6 m/s. Suppose that the uncertainty Δv in the speed is 0.2×10^6 m/s (10% of the speed). What is the minimum uncertainty in the position of the electron?

$$\Delta x\, \Delta p \geq \frac{h}{2\pi}$$

$$\Delta p = m\Delta v = 1.82 \times 10^{-25} \text{ kg} \cdot \text{m/s},$$

$$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s},$$

$$\Delta x \geq \frac{6.63}{6.28} \times \frac{10^{-34}\ \text{J} \cdot \text{s}}{1.82 \times 10^{-25}\ \text{kg} \cdot \text{m/s}},$$

$$\Delta x \geq 5 \times 10^{-10}\ \text{m}.$$

The uncertainty in position is of the order of atomic dimensions and is significant in atomic problems. It is impossible to specify exactly where an electron is in an atom.

The reason for the difference between these two results is that Planck's constant h is very small, so small that the uncertainty principle becomes important only on the atomic scale. For ordinary-sized objects, the equations give the same result as if h had the value zero.

CHAPTER 17. PROBING THE NUCLEUS

The Mathematics of Decay

The *activity* of a sample, the number of disintegrations per second, the decay rate are alternative expressions for the same quantity. Using the letter N to represent generally the number of atoms of a given kind present in a radioactive sample, the activity is $\Delta N/\Delta t$, where ΔN is the number of atoms disintegrating in the same interval Δt. But $\Delta N/\Delta t$ depends both on the type of atom involved, and how many happen to be in the sample. Therefore, a more useful quantity is needed. If, in a time interval Δt, ΔN atoms disintegrate out of a total number N, the *fraction* of atoms disintegrating is $\Delta N/\Delta N$. The *fraction* of atoms *disintegrating per unit time is* $\Delta N/N/\Delta t$. (This quantity can be thought of as the ratio of the activity $\Delta N/\Delta t$ to the total number, N.) This quantity, usually called λ or the decay constant, will be important, as you will see at once below. It is analogous to the death rate in a human population. In the United States, for example, about 5,000 persons die each day out of a population of about 200,000,000. The death rate is therefore one person per 40,000 per day (or one person per day per 40,000).

The beautifully simple mathematical aspect of radioactive decay is that the fraction of atoms decaying per second does not change with time. If initially there are N_0 atoms, and a certain fraction λ decay in 1 s, the actual number of atoms decaying in 1 s is λN_0. Then, at any later time t,

when there are only N_t, atoms remaining, the *fraction* that decay in 1 s will still be λ, but the *number* of atoms decaying in 1 s is now λN_t, a smaller number than before.

The constant fraction λ of atoms decaying per unit time is called the *decay constant*. The value of this constant λ can be found for each radioactive species. For example, λ for radium is 1.36×10^{-11} per second, which means that on average 0.0000000000136th of the total number of atoms in any sample of radium will decay in 1 s.

The fact that λ is a constant can be represented by the expression

$$\lambda = \frac{\Delta N/\Delta t}{N} = \text{constant}$$

which can be rewritten as

$$\frac{\Delta N}{\Delta t} = \text{constant} \times N \qquad \text{or} \qquad \frac{\Delta N}{\Delta t} \propto N.$$

This form of the relation expresses clearly the fact that the decay rate depends directly on the number of atoms left.

By using calculus, a relation of this type can be turned into an expression for N as a function of elapsed time t:

$$\frac{N_t}{N_0} = e^{-\lambda t} \qquad \text{or} \qquad N_t = N_0 e^{-\lambda t},$$

where N_0 is the number of atoms at $t = 0$, N_t is the number remaining unchanged at time t, and e is a mathematical constant that is approximately equal to 2.718. The factor $e^{-\lambda t}$ has the value 1 when $t = 0$, and decreases toward 0 as t increases. Since the decay constant appears as an exponent, the decay is called "exponential" and takes the form shown by the graph in Section 17.9.

The relationship between the half-life $T_{1/2}$ and the decay constant λ can be derived as follows. Write the exponential decay equation in logarithmic form by taking the logarithm of both sides of the equation

$$\log \frac{N_t}{N_0} = \log e^{-\lambda t} = -\lambda t \log e.$$

After a time equal to the half-life $T_{1/2}$, the ratio $N_t/N_0 = ½$. So you can substitute ½ for N_t/N_0 if you substitute $T_{1/2}$ for t in the above equation, and get

$$\log (½) = -\lambda T_{1/2} \log e.$$

The value of log (½) is −0.301 and the value of log e = 0.4343; therefore,

$$-0.301 = -\lambda T_{1/2}(0.4343),$$

and

$$\lambda T_{1/2} = 0.693.$$

So the product of the decay constant and the half-life is always equal to 0.693. Knowing either one allows you to compute the other.

For example, radium-226 has a decay constant $\lambda = 1.36 \times 10^{-11}$ per second; so

$$(1.36 \times 10^{-11}\ \text{s}^{-1})T_{1/2} = 0.693,$$

$$T_{1/2} = \frac{0.693}{1.36 \times 10^{-11}\ \text{s}^{-1}},$$

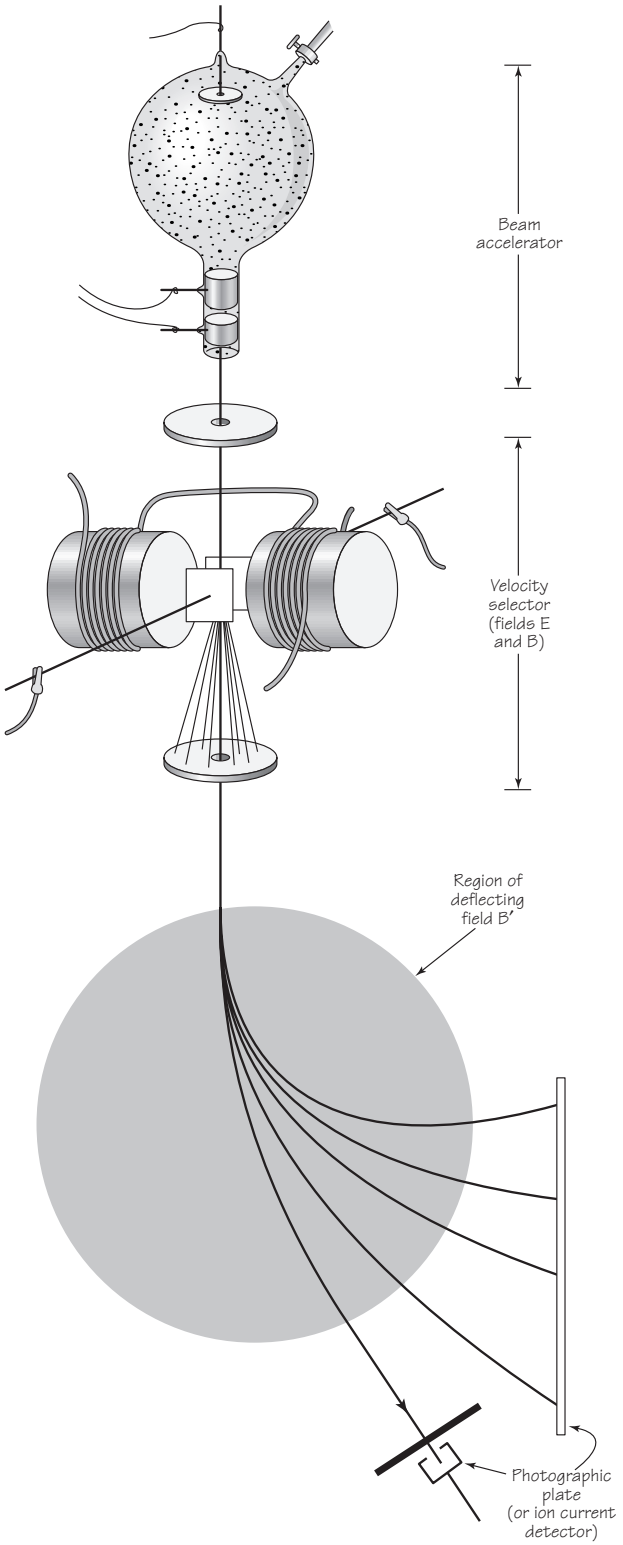
$$T_{1/2} = 5.10 \times 10^{10}\ \text{s}.$$

Thus, the half-life of radium-226 is 5.10×10^{10} s (about 1620 yr).

The Mass Spectrograph

The magnetic separation of isotopes begins by electrically charging the atoms of a sample of material, for example, by means of an electric discharge through a sample of gas. The resulting ions are then further accelerated by means of the electric potential difference between the lower pair of electrodes, and a beam emerges.

Before the different isotopes in the beam are separated, there is usually a preliminary stage that allows only those ions with a certain velocity to pass through. In one type, the ion beam initially enters a region of crossed magnetic fields B and E, produced by current in coils and charged plates as shown. There, each ion experiences a magnetic force of magnitude qvB and an electric force of magnitude qE. The magnetic and electric forces act on an ion in opposite directions, and only for ions of a certain speed will


Beam accelerator
Velocity selector (fields E and B)
Region of deflecting field B′
Photographic plate (or ion current detector)

the forces be balanced, allowing them to pass straight through the crossed fields and the hole in the diaphragm below them. For each of these ions, $qvB = qE$; so their speed $v = E/B$. Because only ions with this speed in the original direction remain in the beam, this portion of the first part of the apparatus is called a *velocity selector*.

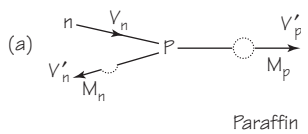
The separation of isotopes in the beam is now accomplished in another magnetic field of strength B'. As the beam enters this field, the magnetic field causes a centripetal force to act on each ion, deflecting it into a circular arc whose radius R depends upon the ion's charge-to-mass ratio. That is, $qvB' = mv^2/r$, and so $q/m = vB'R$.

The divided beams of ions fall on either a photographic plate (in a mass spectro*graph*) or a sensitive ion current detector (in a mass spectro*meter*), allowing the radii R of their deflections to be calculated from the geometry of the apparatus. Since v, B', and R can be determined from measurements, the charge-to-mass ratio of each beam of ions can be calculated directly.

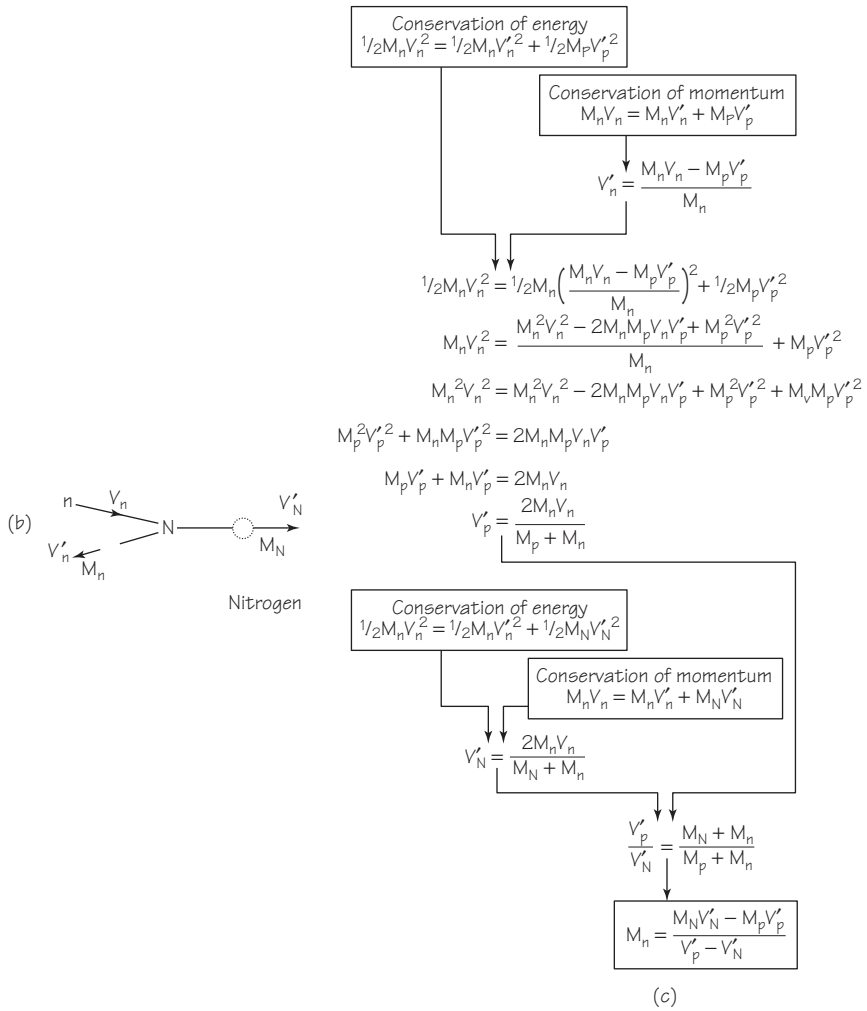
Because this method uses electric and magnetic fields, it is called the *electromagnetic method of separation of isotopes*.

CHAPTER 18. THE NUCLEUS AND ITS APPLICATIONS

Determining the Neutron's Mass



(a) The sketch in (a) represents an elastic collision of a neutron (n) and a proton (p). If it were a head-on collision, the neutron would rebound straight back and the proton would be seen to emerge along the *same* line. To determine the mass of the neutron, m_n, you may use the principles of conservation of kinetic energy and conservation of momentum, which provide two algebraic equations that must both hold. The case is particularly simple if you consider a perfectly elastic head-on collision. As shown in (c), an expression for the proton's recoil speed v'_p can be derived by combining the equations algebraically (solving the momentum equation for v_n, substituting the resulting expression for v'_n in the energy equation, expanding, collecting terms, and solving for v'_p). However, this expression includes the term v_n, the neutron's initial speed, which cannot be measured directly. You can elim-



inate v_n from the equation by analyzing another collision and combining the results with what you already have.

(b) The sketch in (b) represents a perfectly elastic collision between a neutron (n) and a nitrogen nucleus (N). When the collision is head-on, you can write energy and momentum equations similar to what you wrote before, but this time *leading to an expression for the recoil speed of the nitrogen nucleus*, v'_N. This expression also includes the unmeasurable quantity v_n.

(c) The v_p equation and v'_N equation are then combined algebraically (eliminating v_n), and solved for m_n. The expression for m_n now contains only terms that can be measured, so the mass of the neutron, m_n, can be calculated. Note that only the ideas developed for ordinary elastic collisions are used here.