CHAPTER 4

Newton's Unified Theory

4.1 NEWTON AND SEVENTEENTH-CENTURY SCIENCE

Forty-five years passed between the death of Galileo in 1642 and the publication in 1687 of Newton's greatest work, the *Philosophiae Naturalis Principia Mathematica*, or the *Principia* (*Principles*) for short. In those years, major changes occurred in the social organization of scientific studies. The new philosophy of experimental science, applied with enthusiasm and imagination, produced a wealth of new results. Scholars began to work together and to organize scientific societies in Italy, France, and England. One of the most famous, the Royal Society of London for Improving Natural Knowledge, was founded in 1662. Through these societies, scientific experimenters exchanged information, debated new ideas, argued against opponents of the new experimental activities, and published technical papers. Each society sought public support for its work and published studies in widely read scientific journals. Through the societies, scientific activities became well defined, strong, and international.

FIGURE 4.1 Isaac Newton (1642–1727).

This development was part of the general cultural, political, and economic change occurring throughout the sixteenth and seventeenth centuries (1500s and 1600s). Artisans and people of wealth and leisure became involved in scientific studies. Some sought to improve technological methods and products. Others found the study of nature through experiment a new and exciting hobby. However, the availability of money and time, the growing interest in science, and the creation of organizations are not enough to explain the growing success of scientific studies. This rapid growth also depended upon able scientists, well-formulated problems, and good experimental and mathematical tools.

Many well-formulated problems appeared in the writings of Galileo and Kepler. Their studies showed how useful mathematics could be when combined with experimental observation. Furthermore, their works raised exciting new questions. For example, what forces act on the planets and cause the paths that are actually observed? Why do objects fall as they do near the Earth's surface?

Good experimental and mathematical tools were also becoming available in that era. As scientists applied mathematics to physics, studies in each field stimulated development in the others. Similarly, the instrument maker and the scientist aided each other. Another factor of great importance was the rapid buildup of scientific knowledge itself. From the time of Galileo, scientists had reported repeatable experiments in books and journals. Theories could now be tested, modified, and applied. Each new study built upon those done previously.

Newton, who lived during this bustling new scientific age, is the central person in this chapter. However, in science as in any other field, many workers made useful contributions. The structure of science depends not only upon recognized geniuses, but also upon many lesser-known scientists. As Ernest Rutherford, one of the founders of twentieth-century nuclear physics, once said (updating his language slightly):

> It is not in the nature of things for any one person to make a sudden violent discovery; science goes step by step, and every person depends upon the work of his or her predecessors. . . . Scientists are not dependent on the ideas of a single person, but on the combined wisdom of thousands. . . .

In order to tell the story properly, we should trace each scientist's debt to others who worked previously and in the same age, and we should trace each scientist's influence upon future scientists. Within the space available, we can only briefly hint at these relationships.

4.2 ISAAC NEWTON

Isaac Newton was born on Christmas Day, 1642, according to the Julian calendar—4 January 1643, according to the Gregorian calendar now in use—in the small English village of Woolsthorpe in Lincolnshire, north of Cambridge. His father had died before his birth, and Isaac, initially a weak infant, then a quiet farm boy, became very dependent upon his mother. He never married. Like young Galileo, Newton loved to build mechanical gadgets and seemed to have a liking for mathematics. With financial help from an uncle, he went to Trinity College of Cambridge University in 1661, where he enrolled in the study of mathematics and was a successful student.

In 1665, the Black Plague swept through England. The officials closed the college, and Newton went home to Woolsthorpe. Drawing upon what he had learned and his own independent work, there he made spectacular discoveries. In mathematics, he developed the binomial theorem and differential calculus. In optics, he worked out a theory of colors. In mechanics, he had already formulated a clear concept of the first two laws of mo-

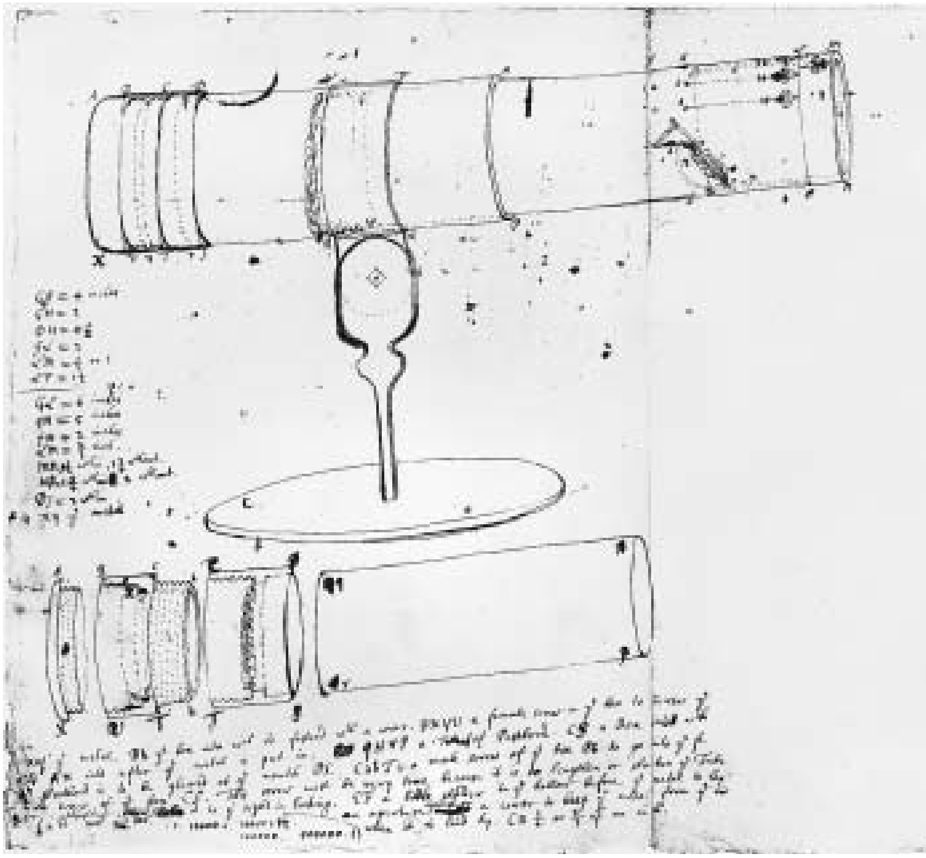


FIGURE 4.2 Newton's drawing of his telescope, made while he was still a student.

tion and the law of gravitational attraction. He also discovered the equation for centripetal acceleration. However, Newton did not announce this equation until many years after Christiaan Huygens' equivalent statement.

This period at Woolsthorpe must have been the time of the famous, though still disputed, fall of the apple. (An ancient apple tree, said to be a descendant of the one from Newton's time, is still on the grounds of the Woolsthorpe home.) One version of the apple story appears in a biography of Newton, written by his friend William Stukeley. In it we read that on a particular occasion Stukeley was having tea with Newton. They were sitting under some apple trees in a garden, and (wrote Stukeley) Newton said that

> he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in a contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the Earth's centre?

The main emphasis in this story probably should be placed on the "contemplative mood" and not on the apple. Newton himself later wrote that he had begun at that time to think of the force on the apple as a force that extends out to the Moon, and that this same type of force might also act between the planets and the Sun. "All this was in the two plague years of 1665 and 1666," he later wrote, "for in those days [at age 21 or 22] I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since." You have seen this pattern before: A great puzzle (here, that of the forces acting on planets) begins to be solved when a clear-thinking person contemplates a familiar event (here the fall of an object on Earth). Where others had seen no relationship, Newton did.

Soon after he returned to Cambridge, Newton succeeded his former teacher as professor of mathematics. Newton taught at the university and contributed papers to the Royal Society. At first his contributions were mainly on optics. His *Theory of Light and Colors*, published in 1672, fired a long and bitter controversy with other scientists who disagreed with his work. Newton, a private and complex man, resolved never to publish again.

In 1684, Newton's devoted friend Edmund Halley, a noted astronomer who later discovered the comet named after him, came to ask Newton's advice. Halley was involved in a discussion with Christopher Wren and Robert Hooke about the force needed to cause a body to move along an ellipse in accord with Kepler's laws. This was one of the most debated and interesting scientific problems of the time. Halley was pleasantly surprised to learn that Newton had already solved this problem "and much other matter," Halley wrote. Halley persuaded his friend to publish these important studies. To encourage Newton, Halley took on responsibility for all the costs of publication. Less than 2 years later, Newton had the *Principia* ready for the printer. Publication of the *Principia* in 1687 quickly established Newton as one of the greatest thinkers in history.

Several years afterward, Newton apparently suffered a nervous breakdown. He recovered, but from then until his death 35 years later, Newton made no major scientific discoveries. He rounded out his earlier studies on heat, optics, chemistry, and electricity, and turned increasingly to theological studies and politics. During those years, he received many honors. In 1699 Newton was appointed Master of the Mint, partly because of his great knowledge of the chemistry of metals. In this position, he helped to reestablish the value of British coins, in which lead and copper had been introduced in place of silver and gold. In 1689 and 1701 Newton represented Cambridge University in Parliament. Queen Anne knighted Newton in 1705. He was president of the Royal Society from 1703 until his death in 1727. Newton is buried in Westminster Abbey.

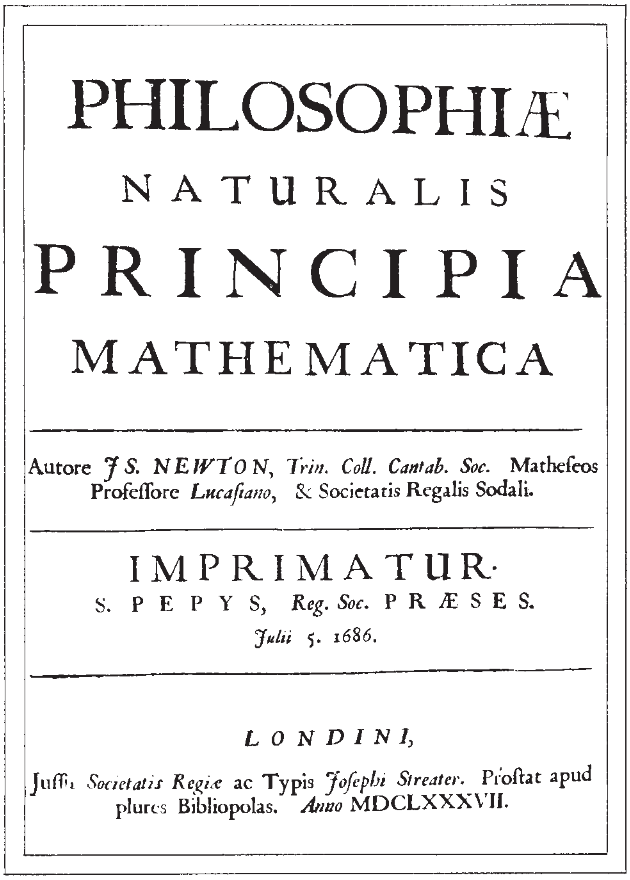
4.3 NEWTON'S *PRINCIPIA*

Newton's *Principia*, written in scholarly Latin, contained long geometrical arguments, and is very difficult to read, even today in English translation. Happily, as was the case later with the relativity and quantum theories, several gifted writers wrote excellent nontechnical summaries in the vernacular languages that allowed a wide circle of educated readers to learn of Newton's arguments and conclusions. The French philosopher Voltaire published one of the most popular of these books in 1736.

The *Principia* begins with the definitions of mass, momentum, inertia, and force. Next come the three laws of motion and the principles of addition for forces and velocities. In a later edition of the *Principia* Newton also included a remarkable passage on "Rules of Reasoning in Philosophy." The four rules, or assumptions, reflect Newton's profound faith in the uniformity of all nature. Newton intended the rules to guide scientists in making hypotheses. He also wanted to make clear to the reader his own philosophical assumptions. These rules had their roots in ancient Greece and are still useful today. The first has been called a principle of parsimony (simplicity or economy), the second and third, principles of unity. The fourth rule expresses a faith needed to use the process of logic.

In a brief form, and using some modern language, Newton's four rules of reasoning are:

1. "*Nature does nothing . . . in vain, and more is in vain when less will serve.*" In short, nature is simple. Therefore, scientists ought not to introduce more hypotheses than are needed to explain observed facts. This fundamental faith of all scientists has a long history, going back to Aristotle, Medieval thought, and Galileo. An example: if a falling apple and the orbiting Moon can be explained by the hypothesis of one force, there is no need for two separate forces, one celestial and one terrestrial.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J S.* NEWTON, *Trin. Coll. Cantab. Soc.* Mathefeos Profeffore *Lucafiano*, & Societatis Regalis Sodali.

IMPRIMATUR·
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Juffu *Societatis Regiæ* ac Typis *Jofephi Streater*. Proftat apud plures Bibliopolas. *Anno* MDCLXXXVII.

FIGURE 4.3 Title page of Newton's *Principia Mathematica*. The Royal Society sponsored the book, and the title page therefore includes the name of the Society's president, Samuel Pepys (famous for his diary describing life during the seventeenth century).

2. *"Therefore to the same natural effects we must, as far as possible, assign the same causes."* For instance, we can assume, at least initially, that the force attracting the Moon to the Earth is the same type of force attracting the Earth and planets to the Sun.
3. *Properties common to all bodies within reach of experiments are assumed (until proved otherwise) to apply to all bodies in general.* For example, all physical objects known to experimenters had always been found to have mass. So, by this rule, Newton proposed—until proved otherwise by new information—that *every* physical object be considered to have mass, even those objects beyond our reach in the celestial region.
4. *In "experimental philosophy" scientists should accept hypotheses or generalizations based on experimental evidence as being "accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined."* That is, scientists should accept such experimentally based hypotheses until

they have additional evidence by which the hypotheses may be made more accurate or revised.

The *Principia* is an extraordinary document. Its three main sections contain a wealth of mathematical and physical discoveries. Overshadowing everything else is the theory of universal gravitation, with the geometrical proofs and arguments leading to it. We shall now restate many of the steps Newton used in his proofs in modern terms in order to make them more accessible.

The central idea of universal gravitation is breathtaking in its generality but can be simply stated: *Every object in the Universe attracts every other object*. Moreover, the amount of attraction in every instance depends in a simple way on the masses of the objects and the distance between them. (We shall patiently develop the steps taken toward finding the equation in Section 4.8, which, in a few symbols, expresses that grand law.)

Newton's great synthesis was boldly to combine terrestrial laws of force and motion with astronomical laws of motion. Gravitation is a *universal* force. It applies to the Earth and apples, to the Sun and planets, and to all other bodies (such as comets) moving in the solar system. Celestial and terrestrial physics were united in one grand system dominated by the law of universal gravitation. The general astonishment and awe were reflected in the famous words of the English poet Alexander Pope:

Nature and Nature's laws lay hid in night:
God said, Let Newton be! and all was light.

Readers of Newton's work must have been excited and perhaps puzzled by the new approach and assumptions. For 2000 years, from the time of the ancient Greeks until well after Copernicus, people believed that the world is separated into two distinct realms, the celestial and the terrestrial. They had used the ideas of natural place and natural motion to explain the general position and movements of the planets. From the time of the Greeks, scholars had widely believed that the planets' orbits were their "natural motion." Therefore, their orbital motion required no explanation; it was natural. However, to Newton the natural motion of a body was continuing to go at a uniform rate along a straight line, in the absence of forces. Motion in a curve showed that a net force was continuously accelerating the planets away from their natural straight-line motion, causing them to move in an orbit. The force acting on the planets was entirely natural and acted between all bodies in heaven and on Earth. Furthermore, it was the same force that caused bodies on the Earth to fall. What a reversal of the old assumptions about what was "natural"!

4.4 THE INVERSE-SQUARE LAW

Newton believed that the influence of the Sun forced the natural straight-line path of a planet into a curve. He demonstrated that Kepler's law of areas could be true if, and only if, forces exerted on the planets are always directed toward a single point. Such a force is termed a *central force*. Newton also showed that the single point is the location of the Sun. Planets obey the law of areas no matter what *magnitude* the force has, as long as the force is always directed to the same point. Newton still had to show that a central gravitational force would cause the exact relationship observed between the orbital radii and the periods of the planets, as given by Kepler's third law. How great was the gravitational force and how did it differ for different planets?

The combination of Kepler's three laws of planetary motion with Newton's three laws of motion in general provides a fine example of the power of logical reasoning. Compare these different sets of laws (you may want to review Section 2.10 and Chapter 3):

Newton's Laws of Motion

1. Every object continues in its state of rest or of uniform velocity (motion at uniform speed in a straight line) unless acted upon by an unbalanced force (a net force). Conversely, if an object is at rest or in motion with uniform velocity, all forces that may be acting on it must cancel so that the net force is zero.
2. The net force acting on an object is numerically equal to, and in the same direction as, the acceleration of the object multiplied by its mass. In symbols: $\mathbf{F}_{net} = m\mathbf{a}$.
3. If one object exerts a force on another object, the second object at the same time exerts a force on the first object. These two forces, each acting on one of the two objects, are equal in magnitude and opposite in direction.

Kepler's Laws of Planetary Motion

1. The planets orbit the Sun on ellipses, with the Sun at one focus and nothing at the other focus.
2. An imaginary line from the Sun to the moving planet sweeps out equal areas in equal amounts of time.
3. The squares of the periods of the planets are proportional to the cubes of their average distances from the Sun, for all of the planets.

Now we will see how these different laws can be put to more general use. According to Newton's first law, a change in motion, either in direction or in magnitude (speed), requires the action of a net force. According to Kepler, the planets move in orbits that are ellipses, that is, curved orbits. Therefore, a net force must be acting to change their motion. Notice that this conclusion alone does not yet specify the type or direction of the net force.

Combining Newton's second law with the first two laws of Kepler clarifies the direction of the force. According to Newton's second law, the net force is exerted in the direction of the observed acceleration. What is the direction of the force acting on the planets? Newton's geometrical analysis indicated that a body moving under a central force will, when viewed from the center of the force, move according to Kepler's law of areas. Kepler's law of areas invokes the distance of the planets from the Sun. Therefore, Newton could conclude that the Sun at one focus of each ellipse was the source of the central force acting on each of the corresponding planets.

Newton then found that motion in an elliptical path would occur only when the central force was an inverse-square force

$$\mathbf{F} \propto \frac{1}{R^2}.$$

Thus, only an inverse-square force exerted by the Sun would result in the observed elliptical orbits described by Kepler. Newton then proved the argument by showing that such an inverse-square law would in fact result in Kepler's third law, the law of periods, $T^2 = kR^3_{\text{av}}$.

From this analysis, Newton concluded that one general law of universal gravitation applied to all bodies moving in the solar system. This is the central argument of Newton's great synthesis.

Early in his thinking, Newton considered the motions of the six then-known planets in terms of their centripetal acceleration toward the Sun. By Newton's proof, mentioned above, this acceleration decreases inversely as the square of the planets' average distances from the Sun. For the spe-

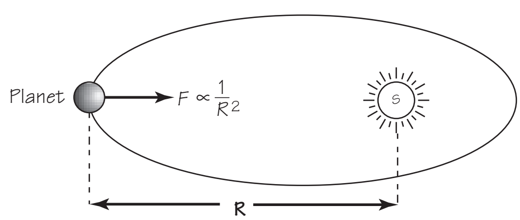


FIGURE 4.4 Planet orbiting Sun, which exerts a force F on the planet that is inversely proportional to the square of the distance to the Sun's center. The ellipse is greatly exaggerated here.

cial case of an ellipse that is just a circle, the proof is very simple and short. The expression for centripetal acceleration a_c of a body moving uniformly in a circular path may be found in terms of the radius R and the period T:

$$a_c = \frac{v^2}{R},$$

$$v = \frac{2\pi R}{T},$$

so

$$a_c = \frac{4\pi^2 R^2}{T^2} \cdot \frac{1}{R} = \frac{4\pi^2 R}{T^2}.$$

Kepler's law of periods stated a definite relationship between the orbital period of every planet and its average distance from the Sun, i.e.,

T^2 is proportional to R^3,

or

$$\frac{T^2}{R^3} = \text{constant}.$$

Using the symbol k for constant

$$T^2 = kR^3_{av}.$$

For circular orbits, R_{av} is just R. Substituting kR^3 for T^2 in the centripetal force equation gives

$$a_c = \frac{4\pi^2 R}{kR^3} = \frac{4\pi^2}{kR^2}.$$

Since $4\pi^2/k$ is also a constant, we have

$$a_c \propto \frac{1}{R^2}.$$

As you can see from the derivation, this conclusion follows necessarily from Kepler's law of periods and the definition of acceleration. If Newton's second law, $\mathbf{F}_{\text{net}} \propto \mathbf{a}$, holds for planets as well as for bodies on Earth, then there must be a centripetal force $\mathbf{F}_c$ acting on a planet. Furthermore, this force must decrease in proportion to the square of the distance of the planet from the Sun

$$F_c \propto \frac{1}{R^2}.$$

Newton showed that the same result holds for all ellipses. Indeed, this proportionality holds for any object moving in an orbit around a center from which the force on the body is applied (even in Bohr's model for hydrogen, in which an electron is thought to orbit around the nucleus). The center of force (in this case the Sun) acts upon any such object by a centripetal force that varies inversely with the square of the distance from the center of force.

Newton had still more evidence from telescopic observations of Jupiter's satellites and Saturn's satellites. The satellites of each planet obeyed Kepler's law of areas around the planet as a center. For Jupiter's satellites, Kepler's law of periods, T^2/R^3 = constant, held. But the *value* of the constant was different from that for the planets around the Sun. The law held also for Saturn's satellites, but with still a different constant. The reason is that Jupiter's satellites were acted on by a central force directed toward Jupiter and decreasing with the square of the distance from Jupiter. The same held true for Saturn's satellites and Saturn. These observed interactions of astronomical bodies supported Newton's proposed central attractive force obeying the $1/R^2$ rule.

4.5 THE LAW OF UNIVERSAL GRAVITATION

Subject to further evidence, you can now accept the idea that a central force is holding the planets in their orbits. Furthermore, the strength of this central force changes inversely with the square of the distance from the Sun. This strongly suggests that the Sun is the origin of the force, but it does not necessarily require this conclusion. Newton's results so far describe the force in mathematical terms, but they do not provide any mechanism for its transmission.

The French philosopher Descartes (1596–1650) had proposed that all space was filled with a thin, invisible fluid. This ethereal fluid carried the planets around the Sun in a huge whirlpool-like motion. This interesting idea was widely accepted at the time. However, Newton proved by a pre-

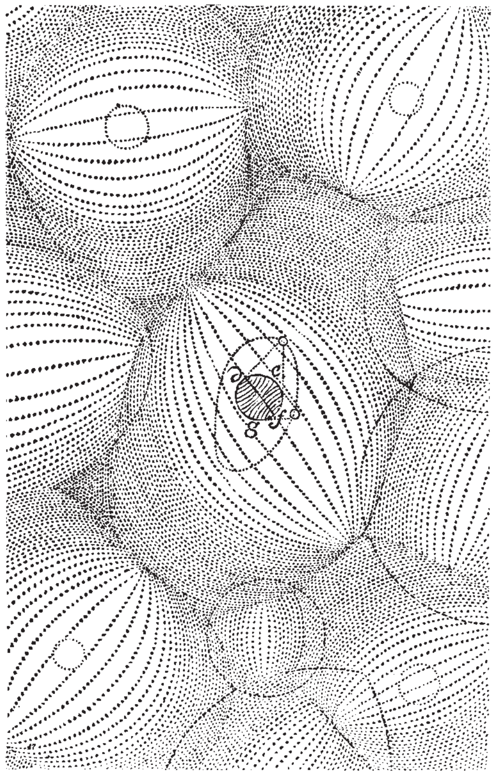


FIGURE 4.5 Drawing by Descartes (1596–1650) illustrating his theory of a space filled with whirlpools of matter that drive the planets along their orbits.

cise argument that this mechanism could not explain the details of planetary motion summarized in Kepler's laws.

Kepler had made a different suggestion some years earlier. He proposed that a magnetic force from the Sun kept the planets moving. Kepler was the first to regard the Sun as the controlling mechanical agent behind planetary motion. But Kepler's magnetic model was inadequate. The problem remained: Was the Sun actually the source of the force? If so, on what properties of the Sun or planets did the amount of the force depend?

As you read in Section 4.2, Newton had begun to think about planetary force while living at home during the Black Plague. There an idea came to him (perhaps when he saw an apple fall). Newton's idea was that the force between the Sun and the planets was the same as the force that caused objects near the Earth's surface to fall. He first tested this idea on the Earth's attraction for the Moon. It was an ingenious argument. The data available to him fixed the distance between the center of the Earth and the center of the Moon at nearly 60 times the radius of the Earth. Newton believed that the attractive force varies as $1/R^2$. Therefore, the gravitational accel-

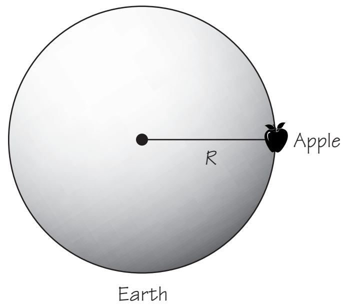


FIGURE 4.6 An apple on Earth's surface.

eration the Earth exerts on the Moon should be only $1/(60)^2 = 1/3600$ of that exerted upon objects at the Earth's surface. Observations of falling bodies had long established gravitational acceleration at the Earth's surface as about 9.80 m/s/s, that is, 9.8 m/s^2. Therefore, the Moon *should* fall at 1/3600 of that acceleration value:

$$9.80 \text{ m/s}^2 \times (1/3600) = 2.72 \times 10^{-3} \text{ m/s}^2.$$

It was also common knowledge that the orbital period of the Moon is very nearly 27.33 days. The centripetal acceleration a_c of a body moving uniformly with period T in a circle of radius r is $a_c = 4\pi^2 R/T^2$, as indicated earlier. When you insert the observed values for the quantities R and T (in meters and seconds) for the case of the Moon orbiting the Earth and do the arithmetic, you find that the centripetal acceleration of the Moon, based upon observed quantities, should be

$$a_c = 2.74 \times 10^{-3} \text{ m/s}^2.$$

This is in very good agreement with the value of 2.72×10^{-3} m/s^2 predicted above. From the values available to Newton, which were close to these, he concluded that he had, in his words,

> compared the force requisite to keep the moon in her orbit with the force of gravity at the surface of the earth, and found them to answer pretty nearly.
>
> Therefore, the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there. And, therefore, (by rules of reasoning 1 and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity. . . .

This was really a triumph. The same gravity that brings apples down from trees also keeps the Moon in its orbit, i.e., making it constantly fall away from a straight-line motion into space, just enough to make it move around the Earth. This assertion is an aspect of what is known as the law of universal gravitation: *Every object in the Universe attracts every other object with a gravitational force*. If this is so, there must be gravitational forces not only between a rock and the Earth, but also between the Earth and the Moon, between Jupiter and its satellites, and between the Sun and each of the planets. And by Newton's third law, we should expect the Moon to exert an equal (but oppositely directed) gravitational force on the Earth, and the planets on the Sun. Indeed that is so, and we shall study this in the next section in detail. But for now, note that because the Earth is so much more massive than the Moon, the "wobble" of the Earth caused by the pull of the Moon is tiny. Similarly, our Sun, enormously more massive than the planets of our solar system, is acted on by them, but this action produces only very small motions of the Sun. This effect has led to a remarkable finding: other stars than our Sun have been observed to exhibit a small wobbling motion that is attributed to the presence of one or more (not directly observable) planets orbiting these stars, thus revealing that other planetary systems exist throughout our Universe.

Newton did not stop at saying that a gravitational force exists between the Earth and Moon, the planets and the Sun. He further claimed that the force is exactly the right magnitude and direction to explain *completely* the motion of every planet. No other mechanism (whirlpools of invisible fluids or magnetic forces) is needed. Gravitation, and gravitation alone, underlies the dynamics of the heavens.

Recently indications have been found that some galaxies experience a repulsion, not predominantly a gravitational attraction, from one another. Hence the expansion of the universe is proceeding at an accelerating rate!

4.6 NEWTON'S SYNTHESIS

The concept of gravitation is now so commonplace that you might be in danger of passing it by without really appreciating what Newton was claiming. First, he proposed a truly universal physical law. Following his rules of reasoning, Newton extended to the whole Universe what he found true for its observable parts. He excluded no object in the Universe from the effect of gravity.

The idea that terrestrial laws and forces are the same as those that regulate the entire Universe had stunning impact. Less than a century before, it would have been dangerous even to suggest such a thing. Kepler and

Galileo had laid the foundation for combining the physics of the heavens and Earth. Newton carried this work to its conclusion. Today, Newton's extension of the mechanics of terrestrial objects to the motion of celestial bodies is called the *Newtonian synthesis*.

Newton's claim that a planet's orbit is determined by the gravitational attraction between it and the Sun had another effect. It moved science away from geometrical explanations and toward physical ones. Most philosophers and scientists before Newton were occupied mainly with the question "What are the motions?" Newton asked, instead, "What force *explains* the motions?" In both the Ptolemaic and Copernican systems, the planets moved about *points* in space rather than about *objects*. The planets moved as they did because of their "nature" or geometrical shape, not because forces acted on them. Newton, on the other hand, spoke not of points, but of things, of objects, of physical bodies. For example, unless the gravitational attraction to the Sun deflected them continuously from straight-line paths, the planets would fly out into the darkness of deep space. Thus, it was the physical Sun that was important, not the point at which the Sun happened to be located.

Newton's synthesis centered on the idea of gravitational force. By calling it a force of gravity, Newton knew that he was not explaining *why* it existed. When you hold a stone above the surface of the Earth and release it, it accelerates to the ground. The laws of motion tell you that there must be a force acting on the stone to accelerate it. You know the *direction* of the force. You can find the *magnitude* of the force by multiplying the mass of the stone by the acceleration. You know that this force is weight, or gravitational attraction to the Earth. But why such an interaction between bodies exists at all remains a puzzle. It is still an important problem in physics today.

4.7 NEWTON AND HYPOTHESES

Newton's claim that there is a mutual force (gravitational interaction) between a planet and the Sun raised a new question: How can a planet and the Sun act upon each other at enormous distances without any visible connections between them? On Earth you can exert a force on an object by pushing it or pulling it. You are not surprised to see a cloud or a balloon drifting across the sky, even though nothing seems to be touching it. Air is invisible, but you know that it is actually a material substance that you can feel when it moves. Objects falling to the Earth, and pieces of iron being attracted to a magnet are harder to explain, but at least the distances involved are small. However, the Earth is over 144 million kilometers, and

Saturn more than 1 billion kilometers, from the Sun. How could there possibly be any physical contact between such distant objects? How can we account for such "action at a distance"?

In Newton's time and for a long time afterward, scholars advanced suggestions for solving this problem. Most solutions involved imagining space to be filled with some invisible substance, called the "ether," which transmitted force. Newton himself privately guessed that such an ether was involved. But he could find no way to test this belief. Therefore, at least in public, he refused to speculate on possible mechanisms. In a famous passage that he added in the second edition of the *Principia* (1713), Newton declared:

> Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. . . . And to us it is enough that gravity does really exist, and acts according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

Newton is quoted at length here because one particular phrase is often taken out of context and misinterpreted. The original Latin reads: *hypotheses non fingo*. This is translated above as "I frame no hypotheses." The sense is, "I do not make untestable, possibly *false*, hypotheses." Newton in fact made many hypotheses in his publications that led to testable results. Also, his letters to friends contain many speculations which he did not publish. So his stern denial of "framing" hypotheses must be properly interpreted.

The fact is that there are two main kinds of hypotheses or assumptions. The most common hypothesis is a proposal of some hidden mechanism to explain observations. For example, you observe the moving hands of a watch. You might propose or imagine some arrangement of gears and springs that causes the motion. This would be a *hypothesis that is directly or indirectly testable, at least in principle, by reference to phenomena*. The hypothesis about the watch, for example, can be tested by opening the watch or by making an X ray film of it. In this context, consider the invisible fluid that supposedly transmitted gravitational force, the so-called "ether." Newton and others thought that certain direct tests might establish the presence of this substance. Many experimenters tried to "catch" the ether. A common approach involved pumping the air from a bottle. Then tests were made to see if any wind, pressure, or friction due to the ether remained to affect objects in the bottle. Nothing of this sort worked (nor has it since).

So Newton wisely avoided making public any hypothesis for which he could not also propose a test.

A quite different type of assumption is often made in published scientific work. It involves a hypothesis which everyone knows is not directly testable, but which still is necessary just *to get started on one's work*. An example is such a statement as "nature is simple," or any other of Newton's four rules of reasoning. Acceptance of either the heliocentric system or the geocentric system is another example. In choosing the heliocentric system, Copernicus, Kepler, and Galileo made the hypothesis that the Sun is at the center of the Universe. They knew that this hypothesis was not directly testable and that either system seemed to explain the observed behavior of the celestial objects equally well. Yet they adopted the point of view that seemed to them simpler, more convincing, and more "pleasing to the mind." It was this kind of initial hypothesis that Newton used without apology in his published work, and it turned out to be right to do so since it did lead to testable results in the end.

Every scientist's work involves both kinds of hypothesis. One popular image of the scientist is of a person who uses only deliberate, logical, objective thoughts, and immediately tests them by definitive experiments. But, in fact, the working scientist feels quite free to entertain any guess, imaginative speculation, or hunch, provable or not, that might be helpful in the early stages of research. (Sometimes these hunches are dignified by the phrase "working hypotheses." Without them there would be little progress!) As Einstein once said, the initial ideas and concepts used in starting research may be considered "free conventions. They appear to be a priori only insofar as thinking without the positing of categories and of concepts in general would be as impossible as is breathing in a vacuum." However, most scientists today do not like to publish something that is still only an unproven hunch. It has to prove its usefulness to be accepted in the final theory.

4.8 THE MAGNITUDE OF THE GRAVITATIONAL FORCE

The general statement that gravitational forces exist universally must now be turned into a quantitative law, as indeed Newton did. An expression is needed for both the *magnitude* and the *direction* of the forces any two objects exert on each other. It was not enough for Newton to assert that a mutual gravitational attraction exists between the Sun and Jupiter. To be convincing, he had to specify what quantitative factors determine the mag-

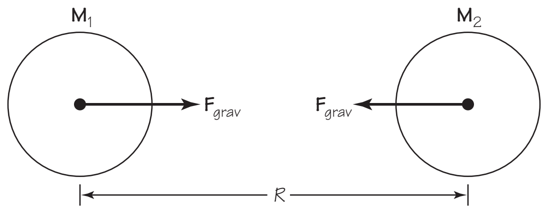


FIGURE 4.7 Mutual gravitational forces.

nitudes of those mutual forces. He had to show how they could be measured, either directly or indirectly.

The first problem was defining precisely the distance *R*. Should it, for example, be taken as the distance between the surface of the Earth and the surface of the Moon? For many astronomical problems, the sizes of the interacting bodies are extremely small compared to the distances between them. In such cases, the distance between the surfaces is practically the same as the distance between the centers. (For the Earth and the Moon, the distance between centers is only about 2% greater than the distance between surfaces.) Yet, some historians believe Newton's uncertainty about a proper answer to this problem led him to drop the study for many years.

Eventually, Newton solved the problem. His calculation showed that the gravitational force exerted *by* a spherical body is the same as if all its mass were concentrated at its center. The gravitational force exerted *on* a spherical body by another body is the same as would be exerted on it if all its mass were concentrated at its center. Therefore, *the distance* R *in the law of gravitation is the distance between centers*.

This was a very important discovery. The gravitational attraction between spherical bodies can be considered as though their masses were concentrated at single points. Thus, in thought, the objects can be replaced by *mass points*.

Newton's third law states that action equals reaction. If this is universally true, the amount of force the Sun exerts on a planet must exactly equal the amount of force the planet exerts on the Sun. For such a very large mass and such a relatively small mass, this may seem contrary to common sense. But the equality is easy to prove. First, assume only that Newton's third law holds between small pieces of matter. For example, a 1-kg piece of Jupiter pulls *on* a 1-kg piece of the Sun as much as it is pulled *by* it. Now consider the total attraction between Jupiter and the Sun, whose mass is about 1000 times greater than Jupiter's. You can consider the Sun as a globe containing about 1000 Jupiters. Define one unit of force as the force that two Jupiter-sized masses exert on each other when separated by the distance of Jupiter from the Sun. Then Jupiter pulls on the *Sun* (a globe of 1000 Jupiters) with a total force of 1000 units. Each of the 1000 parts of the Sun also pulls *on* the planet Jupiter with 1 unit. Therefore, the total pull of the Sun on Jupiter is also 1000 units. Each part of the massive Sun not only pulls *on* the planet, but is also pulled upon *by* the planet. The more mass there is to *attract*, the more there is to be *attracted*. But note: Although the mutual attractive forces on the two bodies are equal in magnitude, the resulting *accelerations* of these are not. Jupiter pulls on the Sun as hard as the Sun pulls on Jupiter, but the Sun *responds* to the pull with only 1/1000 of the acceleration, because its *inertia* is 1000 times Jupiter's.

We saw earlier why bodies of different mass fall with the same acceleration near the earth's surface. The greater the inertia of a body, the more strongly it is acted upon by gravity; that is, near the Earth's surface, the gravitational force on a body is directly proportional to its mass. Like Newton, extend this earthly effect to all gravitation. You then can assume that the gravitational force exerted on a planet by the *Sun* is proportional to the mass of the planet. Similarly, the gravitational force exerted on the Sun by the *planet* is proportional to the mass of the Sun. You have just seen that the forces the sun and planet exert on each other are equal in magnitude (though opposite in direction). It follows that the magnitude of the gravitational force is proportional to the mass of the Sun *and* to the mass of the planet; that is, the gravitational attraction between two bodies is proportional to the *product* of their masses. That is, if the mass of either body is tripled, the force each experiences is tripled. If the masses of both bodies are tripled, the force is increased by a factor of 9. Using the symbol F_{grav} for the magnitude of the forces

$$F_{\text{grav}} \propto m_{\text{planet}}\, m_{\text{Sun}}.$$

The conclusion is that the amount of attraction between the Sun and a planet is proportional to the product of their masses. Earlier you saw that the gravitational attraction also depends on the square of the distance between the centers of the bodies.

$$F_{\text{grav}} \propto \frac{1}{R^2}.$$

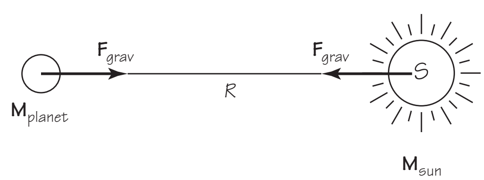


FIGURE 4.8 Gravitational attraction between a planet and the Sun.

Combining these two proportionalities gives *one* force law, which now includes both mass and distance

$$F_{\text{grav}} \propto \frac{m_{\text{planet}}\, m_{\text{Sun}}}{R^2}.$$

Such a proportionality can be written as an equation by introducing a constant. (The constant allows for the units of measurement used.) Using G as a symbol of the proportionality constant, the law of planetary forces can be written as an equation

$$F_{\text{grav}} = \frac{G m_{\text{planet}}\, m_{\text{Sun}}}{R^2}.$$

This equation asserts that the force between the Sun and any planet depends *only* upon three factors. These factors are the masses of the Sun and planet and the distance between them. The equation seems unbelievably simple when you remember how complex the observed planetary motions seemed. Yet every one of Kepler's empirical laws of planetary motion agrees with this relation. In fact, we can *derive* Kepler's empirical laws from this force law and Newton's second law of motion. More important still, details of planetary motion not obtainable with Kepler's laws alone can be calculated using this force law.

Newton's proposal that this simple equation describes completely the forces between the Sun and planets was not the final step. He saw nothing to restrict this mutual force to the Sun and planets, or to the Earth and apples. Rather, again in line with his rules of reasoning, Newton insisted that an identical relation should apply *universally* to all matter everywhere throughout the Universe. This relation would hold true for *any two bodies* separated by a distance that is large compared to their dimensions. It would apply equally to two atoms or two stars. In short, Newton proposed a *general law of universal gravitation*

$$\boxed{F_{\text{grav}} = \frac{G \cdot m_1 m_2}{R^2}}$$

Here m_1 and m_2 are the masses of the two bodies and R is the distance between their centers. The numerical constant G is called the *constant of universal gravitation*. Newton assumed it to be the same for all gravitational interactions, whether between two grains of sand, two members of a solar system, or two stars in different parts of the sky. As you will see, the very great successes made possible by this simple relationship have borne out Newton's assumption. In fact, scientists have come to assume that this equation applies everywhere and at all times, past, present, and future.

Even before we consider more supporting evidence, the sweeping majesty of Newton's law of universal gravitation should command your wonder and admiration. It also leads to the question of how such a bold universal law can be proved. There is no complete proof, of course, for that would mean examining every interaction between all bodies everywhere in the Universe! But the greater the variety of individual tests made, the greater will be our belief in the validity of the law.

4.9 THE VALUE OF *G*, AND SOME CONSEQUENCES

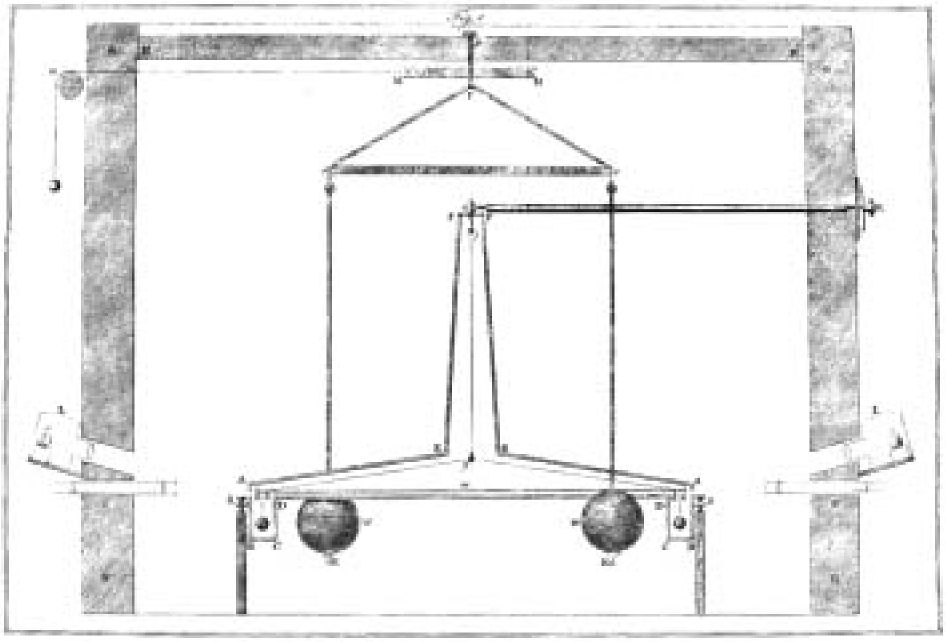
The masses of small solid objects on the surface of the Earth can be found easily enough from their weights. Measuring the distance between solid objects of spherical shape presents no problem. But how can one measure the tiny mutual gravitational force between relatively small objects in a laboratory? (Remember that each object is also experiencing separately a huge gravitational force toward the tremendously massive Earth.) In addition, how can we find the actual value of G?

This serious technical problem was eventually solved by the English scientist, Henry Cavendish (1731–1810). For measuring gravitational forces, he employed a torsion balance. In this device, the gravitational attraction between two pairs of lead spheres, one pair fixed in the laboratory, the other suspended from a wire on a rod holding up that pair (see the illustration). The force producing a twist of the wire could be calibrated by applying first to the suspended pair of lead spheres small known forces. A typical experiment might test the attraction between one of the fixed spheres of, say, 100 kg and one of the suspended spheres of, say, 1 kg, at a center-to-center distance of 0.1 m. The resulting force would be found to be about one-millionth of a newton (0.000001 N)! These data, inserted into the force law of universal gravitation, allow one to calculate a value for G; it comes out to be about 10^{-10} (N m^2/kg^2). The experiment to find the value of G has been steadily improved ever since, and the accepted value of G is now

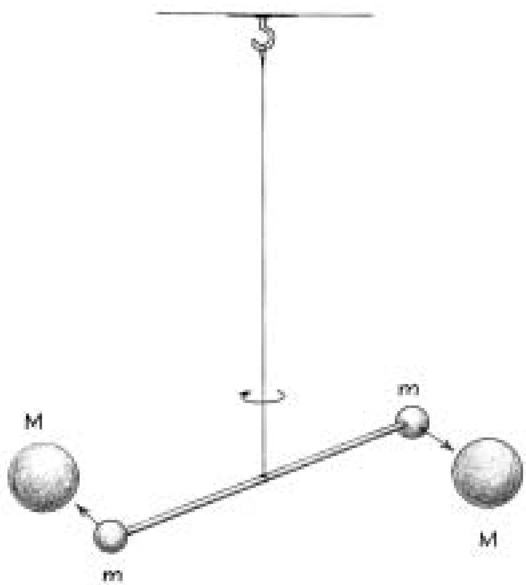
$$G = 0.00000000006673 \text{ N m}^2/\text{kg}^2.$$

This can be written more concisely in scientific notation:

$$G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2.$$



(a)



(b)

FIGURE 4.9 (a) Schematic diagram of the device used by Cavendish for determining the value of the gravitational constant *G*. Large lead balls of masses M_1 and M_2 were brought close to small lead balls of masses m_1 and m_2. The mutual gravitational attraction between M_1 and m_1 and between M_2 and m_2 caused the vertical wire to be twisted by a measurable amount. (b) Diagram of Cavendish's apparatus for determining the value of *G*. To prevent disturbance from air currents, Cavendish enclosed the apparatus in a sealed case and observed the deflection of the balance rod from outside with telescopes.

Using this result in the formula giving the law of universal gravitation allows one to calculate the force in newtons (N), if the distance R between the centers of the attracting objects is measured in meters (m) and the masses of the objects are measured in kilograms (kg).

G is obviously a very small number. The measurement of G confirms that the gravitational force between everyday objects is indeed very small. Let's see how small. Suppose you have two 1-kg masses separated by a distance of 1 m between their centers. How large is the gravitational attraction between them? To find out, simply substitute into the equation for F_{grav}:

$$F_{grav} = \frac{Gm_1m_2}{R^2}$$

$$= (6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1 \text{ kg} \cdot 1 \text{ kg}/1 \text{ m}^2)$$

$$= 6.67 \times 10^{-11} \text{ N}$$

or

$$F_{grav} = 0.0000000000667 \text{ N}.$$

Obviously this is an extremely small force. Could you ever measure a force to this accuracy in your laboratory? For comparison, it turns out that an average apple (mass of about 0.1 kg) weighs (has the gravitational force on it of) about 1 N—which is only right!

The Acceleration of Gravity *g*

You know from the previous chapter that all objects on Earth fall at the same rate of acceleration, regardless of their mass. Since this rate of acceleration is nearly the same everywhere on Earth, it is given a special symbol, g. Using Newton's law of gravitation, and the value of G, how can we explain that g is nearly constant over the entire Earth's surface? Consider an apple of mass m. From the previous chapter, the gravitational force on the apple at the Earth's surface is

$$F_{grav} = m \cdot a = m \cdot g.$$

We found in this chapter that the gravitational force is given also by Newton's equation

$$F_{grav} = G \cdot \frac{M_{Earth}\, m}{R^2_{Earth}}.$$

Therefore, these two expressions must be equal to each other

$$G \cdot \frac{M_{\text{Earth}}\, m}{R^2_{\text{Earth}}} = m \cdot g.$$

Canceling the mass of the apple (m) on both sides of the last equation, we find that g is related to the mass and radius of the Earth

$$G \cdot \frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} = g.$$

This equation helps to explain why the acceleration of gravity g is nearly the same for all objects on the Earth, regardless of their own mass: The value of g is determined by the universal constant G, the mass of the Earth M_{Earth}, which is the same in each case, and the radius of the Earth, R_{Earth}, also near enough the same in each case.

To be sure, g is not *exactly* the same everywhere on the Earth's surface, because the distance to the center of the Earth and its local mass density are slightly different at different places on the surface of the Earth. The Earth is not a perfect, homogeneous sphere. Nevertheless, to one decimal place, the result for g is near enough the same everywhere on the Earth. (For example, at the top of Mount Everest the value of g is 9.7647 m/s^2, while in Toronto, Canada, it is 9.8049 m/s^2.) This means that the gravitational force on objects near the Earth's surface, also known as the weight, is different for each mass; but the acceleration (g) will be about the same for all.

Of course, the acceleration due to gravity will be different from the acceleration on Earth if the attracting object is not the Earth. For instance, on the surface of the Moon, which has a smaller radius and a smaller mass than the Earth's, the acceleration due to gravity is only 1.6 m/s^2. Thus, an astronaut having a mass of 70 kg, would weigh 690 N on the surface of the Earth, but only 110 N on the Moon.

The law of gravitation may also be used to "weigh the Earth," i.e., to find its mass. This amazing calculation may be found in the *Student Guide*.

Revisiting the Space Shuttle

In the previous chapter (Section 3.12) we looked at the flight of the space shuttle *Endeavor* as it orbited the Earth in January 1996. Data available on the NASA Web site indicated that the shuttle flew in a nearly circular orbit of 288 miles average altitude above the Earth's surface with a period of

AVIATION TECHNOLOGY

The English physicist George Cayley worked out the essentials of aerodynamic theory in 1799. However, it would take another one hundred years before aviation technology would be ready for the flight of a heavier-than-air machine that seemed to defy gravitation (which it did not). For the next century those studying aerodynamics focused on finding the working ratio between the weight of the craft and the power required to make it fly. Wilbur and Orville Wright, two bicycle manufacturers from Dayton, Ohio, were the first to solve this problem successfully. On December 17, 1903, Orville Wright became the first man ever to fly an airplane, using a 12-horsepower engine to fly a biplane with a 40-ft 4-in wingspan.

FIGURE 4.10 Orville Wright (in prone position) piloting Flyer, December 17, 1903.

After World War I commercial airlines began to be formed in order to harness the increasing number of aviation inventions. In contrast to European government subsidies, the 1920s American government had no coherent national policy concerning the production of aircraft or the operation of airlines. As a result many private American airline companies were created during the 1920s but very few of them lasted more than a couple of years before going out of business. The most successful American airline during this period was Pan American Airlines, whose founder Juan Trippe concentrated on routes to Latin America.

Two central problems still remained to be overcome, however, if air flight was going to challenge the supremacy of rail travel, particularly in America. These problems were safe high-altitude flying and safe night flying. In 1930, the airplane manufacturer Boeing responded to this need by designing and producing the Monomail, which was an all-metal plane with retractable landing equipment. This evolved into the 247 model in 1933, which could cruise at 155 mi/hr carrying ten passengers, twice as fast as its competitors.

Airline operators and military users of these planes, however, wanted still bigger and faster craft. American manufacturers had begun tinkering with various ways to improve the piston engines: in particular, they had been experimenting with turbochargers during the previous decade.

In a turbocharger, exhaust gases from the piston engine drive the turbine. The turbine drives a supercharger, which acts as an air compressor, delivering additional air to run the engine.

The aim of the turbocharger was to maintain power at high altitudes, by compressing the air that goes into the piston engine. The compressor would be driven by a turbine placed in the hot engine exhaust whose gases carried so much power that this system was extremely effective, and cost free. However, regardless of this simplicity and the cost effectiveness of this design, the turbocharger did not attract investors within the aviation industry, largely because the temperatures within the turbine were too high for the steel that was used in its construction.

The next major improvement in aircraft engines was the turbojet; which was developed in Germany by Max Hahn and Hans von Ohain and in Britain by Frank Whittle. The main benefit of the turbojet was that its light weight offered a favorable weight to power ratio. The American company Boeing became one of the first successfully to design and manufacture a passenger plane powered by a turbojet. They introduced their 707 model onto the market in 1958, and it was so economical that it was soon followed by the Boeing 727 that could carry 100 passengers, and therefore rivaled the biggest piston planes on the market. When the Boeing 747, known as the "jumbo jet," came onto the market in 1970, it could carry 500 passengers, and still have room for cargo, freight, and mail.

Further Reading

T.A. Heppenheimer, *Turbulent Skies: The History of Commercial Aviation* (New York: Wiley, 1998).

5425 seconds for each orbit. From these data we calculated that the shuttle exhibited a centripetal acceleration of

$$a_c = 9.2 \text{ m/s}^2$$

which is about 13.6% less than the gravitational acceleration g at the surface of the Earth. We surmised that the centripetal acceleration on the shuttle was provided by the force of gravity at that altitude. From this we concluded that the gravitational force appears to decrease with distance from the Earth.

Now we can affirm this conclusion on the basis of the law of universal gravitation. Earlier in this section we found that the acceleration of gravity at the distance R from the center of a massive object M is

$$a_g = \frac{GM}{R^2}.$$

This holds if the point where the calculation is measured is outside of the object. If the centripetal acceleration experienced by the shuttle Endeavor does indeed arise from the force of gravity at the shuttle's altitude, then the calculated centripetal acceleration, $a_c = 9.2 \text{ m/s}^2$, should be close to the acceleration of gravity calculated from the law of gravitation. So we may ask which is correct:

$$a_c = a_g$$

or

$$9.2 \text{ m/s}^2 = \frac{GM}{R^2} \text{ ?}$$

In order to answer these questions, we need to substitute the values for G, M, and R in the second equation and see if the result is 9.2 m/s^2. We have now found the value for G and for M, which is the mass of the Earth, $M = 6.0 \times 10^{24}$ kg (to one decimal place). The radius R of the shuttle's orbit, which is assumed to be circular, must be measured from the center of the Earth (not from the surface). We found in Section 3.12 that $R = 6833 \text{ km} = 6.8 \times 10^6$ m (to one decimal place). Substituting, we have

$$\frac{GM}{R^2} = \frac{(6.8 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg})}{(6.8 \times 10^6 \text{ m})^2}$$

$$= \left[\frac{(6.8 \times 10^{-11})(6.0 \times 10^{24})}{(46.2 \times 10^{12})}\right] \frac{\text{N m}^2 \text{ kg}}{\text{m}^2 \text{ kg}^2}$$

$$= 8.8 \text{ m/s}^2.$$

The calculated result of 8.8 m/s^2 is within about 4.3% of the expected acceleration of 9.2 m/s^2. Since we have assumed that the orbit of the shuttle was a perfect circle, which it was not, we can be fairly certain that our result is within the limits of accuracy of this assumption. Therefore our conclusion that the centripetal acceleration on the shuttle is just the force of gravity at that altitude is supported. However, we would have to repeat this calculation again for the flights of other satellites, and perhaps even test it for a satellite in an orbit that is as nearly circular as possible. This is how conclusions based upon experimental evidence are often obtained.

4.10 FURTHER SUCCESSES

Newton did not stop with the fairly direct demonstrations described so far, accessible to him at that time. In the *Principia*, he showed that his law of universal gravitation could explain other complicated gravitational interactions. Among these were the tides of the sea and the peculiar drift of comets across the sky.

The Tides

Knowledge of the tides had been vital to navigators, traders, and explorers through the ages, including of course especially Newton's fellow countrymen, the British traders and explorers. But the *cause of* the tides had remained a mystery despite the studies of such scientists as Galileo. By applying the law of universal gravitation, Newton was able to explain the main features of the ocean tides. He found them to result from the attraction of the Moon and the Sun upon the waters of the Earth. Because the Moon is so much nearer to the Earth, its attractive force on the oceans is greater than the Sun's force on the oceans. Each day, as the Earth rotates, two high tides normally occur. Also, twice each month, at full Moon and at new Moon, the Moon, Sun, and Earth are in line with each other. At these times the tidal changes are greater than average.

Two questions about tidal phenomena demand special attention. First, why do high tides occur on both sides of the Earth, including the side away from the Moon? Second, why does high tide occur at a given location some hours after the Moon is highest in the sky?

Newton knew that the gravitational attractions of the Moon and Sun act not only on the ocean but also on the whole solid Earth. These forces accelerate both the fluid water on the Earth's surface and the Earth itself. Newton realized that the tides result from the *difference* in acceleration of the Earth and its waters. The Moon's distance from the Earth's center is about 60 Earth radii. On the side of the Earth nearer the Moon, the distance of the water from the Moon is only 59 radii. On the side of the Earth away from the Moon, the water is 61 Earth radii from the Moon. (See Figure 4.11.) On the side nearer the Moon, the force on the water toward the

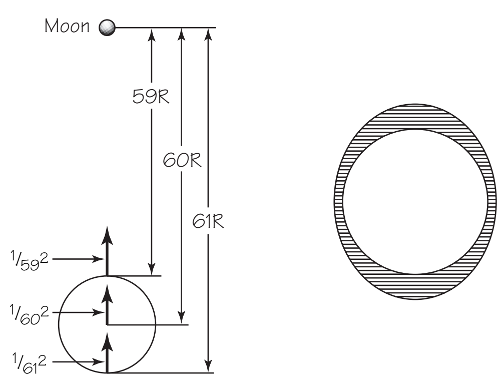


FIGURE 4.11 Relative amounts of tidal forces (note that the Earth-Moon distance indicated in the figure is greatly reduced owing to space limitations).

Moon is slightly greater than the force on the Earth as a whole, because the distance *R* is slightly smaller. The net effect is that the water is pulled away from the Earth. On the side of the Earth away from the Moon, the pull on the water toward the Moon is less than that of the Earth as a whole since its distance to the Moon is slightly greater. The net result is that the Earth is pulled away from the water there. A point on the ocean shore on the Earth therefore experiences two high tides each day. In between, there are low tides because the water is depleted at those points on the globe between the high tides.

Perhaps you have watched the tides change at the seashore or examined tide tables. If so, you know that high tide occurs some hours *after* the Moon is highest in the sky. To understand this, even qualitatively, you must remember that relative to the size of the Earth the oceans are not very deep. The ocean waters moving in from more distant parts of the oceans in response to the Moon's attraction are slowed by friction with the ocean floors, especially in shallow water. Thus, the time of high tide is delayed. In any particular place, the amount of delay and the height of the tides depend greatly upon how easily the waters can flow. No general theory can account for all the particular details of the tides. Most local predictions in the tide tables are based, in part, on empirical rules using the tidal patterns recorded in the past.

Since there are tides in the seas, you may wonder if the atmosphere and the Earth itself undergo tides. They do. The Earth is not completely rigid, but bends somewhat, like steel. The high tide in the Earth's crust is about 30 cm high. The atmospheric tides are generally masked by other weather changes. However, at altitudes of about 160 km, satellites have recorded considerable rises and falls in the thin atmosphere.

Comets

From earliest history through the Middle Ages, people have interpreted comets as omens of disaster. Halley and Newton showed them to be only shiny, cloudy masses moving around the Sun according to Kepler's laws, just as planets do. They found that most comets are visible only when closer to the Sun than the distance of Jupiter. Today we know that they are composed of frozen water and gases, as well as dirt—a "dirty snowball."

Several very bright comets have orbits that take them well inside the orbit of Mercury. Such comets pass within a few million kilometers of the Sun. Many orbits have eccentricities near 1.0; these comets have periods of thousands or even millions of years. Some other faint comets have periods of only 5–10 years. Occasionally comets collide with planets, as happened when the Schoemaker–Levy comet smashed into Jupiter in 1994. An

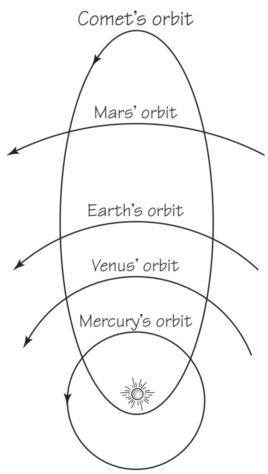


FIGURE 4.12 Schematic diagram of the orbit of a comet in the ecliptic plane. Comet orbits are tilted at all angles.

observation satellite studying Jupiter at that time, appropriately named Galileo, recorded the spectacular effects on the outer gases of Jupiter, providing much valuable information on Jupiter's atmosphere.

Unlike the planets, all of whose orbits lie nearly in a single plane, the planes of comet orbits tilt at all angles. Yet, like all members of the solar system, they obey all the laws of dynamics, including Kepler's laws and the law of universal gravitation.

Edmund Halley (1656–1742) applied Newton's concepts of celestial motion to the motion of bright comets. Among the comets he studied were those that had been observed in 1531, 1607, and 1682. Halley found the orbits for these comets to be very nearly the same. He suspected that all these observations were really of one and the same comet, moving in a closed orbit with a period of about 75 years. He predicted that the comet would return to where people on Earth could see it in about 1757—which it did, although Halley did not live to see it. It was a spectacular verification of Newton's law of gravitation. Halley's comet appeared again in 1833, 1909, and in 1985–86. It is due to return to the vicinity of the Sun in 2061.

With the period of Halley's bright comet known, its dates of appearance could be tracked back in history. Ancient Indian, Chinese, and Japanese documents record all of the expected appearances since 240 B.C. except one. Almost no European records of this great comet exist. This is a sad comment upon the level of culture in Europe during the so-called Dark Ages. One of the few European records is on part of the famous Bayeux tapestry, embroidered with 72 scenes of the Norman Conquest of England in 1066. One scene shows the comet overhead while King Harold of England and his court cower below, taking the appearance to presage a disaster. But with Newtonian science explaining the paths of comets, they came to be seen as regular members of the solar system, instead of unpredictable, fearful events.

Beyond the Solar System

You have seen how Newton's laws explain motions and other physical events on the Earth and in the solar system. Now consider a new and even broader question: Do Newton's laws also apply at greater distances, for example, among the stars?

Over the years following publication of the *Principia*, several sets of observations provided an answer to this important question. One observer was William Herschel, a British musician turned amateur astronomer. In the late 1700s, with the help of his sister Caroline, Herschel made a remarkable series of observations. Using homemade, high-quality telescopes, Herschel hoped to measure the parallax of stars owing to the Earth's motion around the Sun. Occasionally he noticed that one star seemed quite close to another. Of course, this might mean only that two stars happened to lie in the same line of sight. But Herschel suspected that some of these pairs were actually double stars held together by their mutual gravitational attractions.

He continued to observe the directions and distances of one star with respect to the other in such pairs. In some cases, one star moved during a few years through a small arc of a curved path around the other. Other astronomers gathered more information about these so-called *double stars*, far removed from the Sun and planets. Eventually, it was clear that they do move around each other according to Kepler's laws. Therefore, their motions also agree with Newton's law of universal gravitation. Using the same equation as that used above for the planets, see *Student Guide*, astronomers have calculated the masses of these stars. They range from about 0.1 to 50 times the Sun's mass.

Here we stop briefly to note again that the essence of science is that it is not dogmatic, i.e., newly found facts can modify and improve a theory. The-

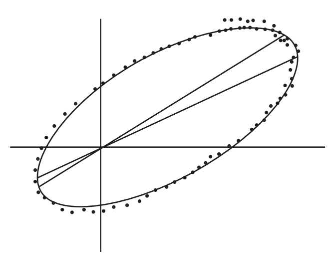


FIGURE 4.13 The motion over many years of one of two components of a binary star system. Each dot indicates the average of observations made over an entire year.

ories become increasingly acceptable as they are found useful over a wider and wider range of problems. No theory has stood this test better than Newton's theory of universal gravitation as applied to the planetary system. It took nearly a century for physicists and astronomers to comprehend, verify, and extend Newton's work on planetary motion. As late as the end of the nineteenth century, most of what had been accomplished in mechanics since Newton's day was but a development or application of his work.

As indicated earlier in reference to Kepler's laws of planetary motion (Section 2.10), a scientific *theory* differs from a *law of nature*, since a theory encompasses data and assumptions and hypotheses that can be altered and improved or abandoned as new experimental data and new ideas become available. A scientific theory is therefore quite different from the common usage of the word "theory," as, for instance, in such an everyday statement as "That's just a theory, which is merely your opinion." On the other hand, a scientific law is a statement about nature. It can be accepted or rejected, and sometimes expanded, but it does not contain hypotheses or assumptions. Examples are Kepler's laws, Newton's laws of motion, and Newton's law of universal gravitation: *Every object in the Universe attracts every other object with a gravitational force given by* $F_{grav} = GM_1M_2/R^2$. Newton's *theory* of gravitation encompasses his law of universal gravitation, but it also encompasses Newton's synthesis of celestial and terrestrial phenomena, as expressed, for instance, in his *Rules of Reasoning*: Not only is there one universal law of gravitation, but it is an expression of the circumstance that there is one universal physics that applies to all physical processes occurring on the Earth and throughout the entire Universe. As we have seen in this chapter, Newton's synthesis and his law of universal gravitation have

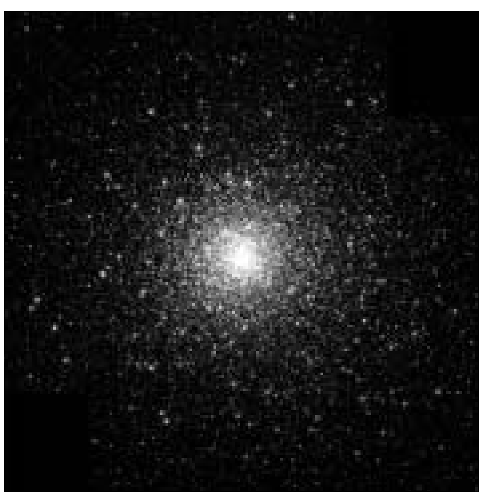


FIGURE 4.14 Globular cluster M80. Globular clusters like this one contain tens of thousands of stars held together by gravitational attraction.

been confirmed many times in many different comparisons with experimental evidence.

4.11 THE NATURE OF NEWTON'S WORK

Today, Newton and his system of mechanics are honored for many reasons. The *Principia* formed the basis for the development of much of our physics and technology. Also, the success of Newton's approach has made it a model for all the physical sciences.

Throughout Newton's work, you will find his basic belief that celestial phenomena can be explained by applying quantitative, earthly laws. Newton felt that his laws had real physical meaning, that they were not just mathematical conveniences behind which unknowable laws lay hidden. The natural physical laws governing the universe *are* accessible through human reason and observation, and they can be expressed in simple mathematical forms of the laws.

Newton combined the skills and approaches of both the experimenter and the theoretician. He invented research equipment, such as the first reflecting telescope. He performed skillful experiments, especially in optics. Yet he also applied his great mathematical and logical powers to the creation of specific, testable predictions.

Like all scientists, Newton also had another weapon: the useful concepts developed by earlier scientists and those of his own time. Galileo and Descartes had contributed the first steps leading to a proper idea of inertia, which became Newton's first law of motion. Kepler's planetary laws were central in Newton's consideration of planetary motions. Huygens, Hooke, and others clarified the concepts of force and acceleration, ideas that had been evolving for centuries.

In addition to his own experiments, Newton selected and used data from a large number of sources. Tycho Brahe was only one of several astronomers whose observations of the motion of the Moon he used. When Newton could not complete his own measurements, he knew whom he could ask or where to look.

Last, recall how completely and how fruitfully he used and expanded his own specific contributions. A good example is his theory of universal gravitation. In developing it, Newton used his laws of motion and his various mathematical inventions again and again. Yet Newton was modest about his achievements. He once said that if he had seen further than others "it was by standing upon the shoulders of Giants."

But, today, scientists recognize that Newton's mechanics, while immensely useful still, holds true only within a well-defined region of science. For example, the forces within each galaxy appear to be Newtonian. But this may not be true for forces acting between one galaxy and another or across the entire universe. In addition to gravitation between galaxies, there appears to be an effect so far little understood but first suspected by Albert Einstein and given the name "dark energy," that seems to overcome that attraction and indeed to accelerate the galaxies away from one another.

At the other end of the scale are atoms and subatomic particles. During the past century, non-Newtonian concepts had to be developed to explain the observed motions of these particles, as will be discussed in Chapter 14.

Even within the solar system, there are several small differences between the predictions based on Newtonian gravitation and the observations. The most famous involves the angular motion of the axis of Mercury's orbit around the Sun. This motion is greater than the value predicted from Newton's laws by about one part in 800 per century. What causes this difference? For a while, it was thought that gravitational force might not vary inversely *exactly* with the square of the distance. One of the greatest triumphs of Einstein's theory of general relativity over 200 years later involved an explanation of this effect, without throwing doubt on the accuracy of Newton's equation for the gravitational force for the cases to which they had been generally applied.

Einstein's success suggests that any perceived difficulties with Newtonian gravitation should not be hastily assigned to an imperfection in the theory as a whole. The law of gravitation applies with unquestionable accuracy to all other planetary motions. But when applied to extreme situations, such as the behavior of a massive object like the planet Mercury moving rapidly in the near vicinity of the Sun, some facets of the theory make it too limited. Many studies have shown that there is no way to modify the details of Newtonian mechanics to explain certain observations in such extreme cases. Instead, these observations can be accounted for only by constructing *new* theories, such as *relativity theory* and *quantum mechanics*, based on some very different assumptions. The predictions from these theories, when applied to ordinary, nonextreme situations, are almost identical to those from Newton's laws for familiar phenomena. But they are accurate also in some extreme cases where the Newtonian predictions alone begin to show inaccuracies. Thus, Newtonian science is linked at one end with *relativity theory*, which is important for bodies with very great mass or moving at very high speeds. At the other end, Newtonian science approaches *quantum mechanics*, which is important for particles of extremely small mass and size, for example, atoms, molecules, and nuclear particles. However, for a vast range of problems between these extremes, Newtonian theory gives accurate results and is far simpler to use. Moreover, it is in Newtonian mechanics that relativity theory and quantum mechanics historically have their roots. (For more on Newton's impact see the *Student Guide* discussion for Chapter 4, "Impact and Reaction.")

SOME NEW IDEAS AND CONCEPTS

action at a distance
ether
gravitation
law of universal gravitation
Newton's synthesis

FURTHER READING

G. Holton and S.G. Brush, *Physics, The Human Adventure* (Piscataway, NJ: Rutgers University Press, 2001), Chapters 12–14, is especially helpful on the structure and method in physical science, as is Chapter 11 on Newton's law of universal gravitation.

D.H. Levy, *More Things in Heaven and Earth: Poets and Astronomers Read the Night Sky* (Wolfville, Nova Scotia: Wombat Press, 1997).

I.B. Cohen, *Science and the Founding Fathers* (New York: Norton, 1995.)

I. Newton, *The Principia: Philosophiae Naturalis Principia Mathematica*, a new translation by I.B. Cohen and A. Whitman, assisted by J. Budenz (Berkeley, CA: University of California Press, 1999).

STUDY GUIDE QUESTIONS

1. Write a brief outline of this chapter, including the main ideas contained in each section.
2. State in your own words Newton's law of universal gravitation.
3. What is Newton's equation for the magnitude of the gravitational force? Carefully define every symbol in this equation.

4.1 Newton and Seventeenth-Century Science

List some characteristics of society during Newton's lifetime that fostered scientific progress.

4.3 Newton's *Principia*

1. Explain Newton's concept of the "whole burden of philosophy," that is, the job of the scientist.

2. In your own words, state Newton's four rules of reasoning and give an example of each.
3. What do these rules have to do with the theory of universal gravitation?
4. State, in your own words, the central idea of universal gravitation.
5. How did Newton differ from Aristotle, who believed that the rules of motion on Earth are different from the rules of motion in the heavens?

4.4 The Inverse-Square Law

1. What can be proved from Kepler's law that the planets sweep out equal areas with respect to the Sun in equal times?
2. With what relationship can Kepler's third law, T^2/R_{av}^3 = constant, be combined to prove that the gravitational attraction varies as $1/R^2$?
3. What simplifying assumption was made in the derivation given in this section?
4. Did Newton limit his own derivation by the same assumption?
5. How did Newton reach the conclusion that one general law of universal gravitation must apply to all bodies moving in the solar system?
6. If two objects are moved twice as far away from one another, by how much is the gravitational force between them decreased? If they are moved three times as far? By how much is the gravitational force increased if the objects are moved together to one-fourth their original separation?
7. While two objects are held at a fixed distance from each other, what happens to the gravitational force if one of the two masses is doubled; if it is tripled; and if both masses are halved?

4.5 The Law of Universal Gravitation

1. What idea came to Newton while he was thinking about falling objects and the Moon's acceleration?
2. Kepler, too, believed that the Sun exerted forces on the planets. How did his view differ from Newton's?
3. What quantitative comparison did Newton make after (supposedly) seeing the apple fall? What was the result?
4. The Moon is 60 times farther from the center of the Earth than objects at the Earth's surface. How much less is the gravitational attraction of the Earth acting on the Moon than on objects at its surface? Express this value as a fraction of 9.8 m/s^2.
5. The central idea of this and the next chapter is the "Newtonian synthesis." What is a synthesis? What did Newton synthesize?

4.6 Newton's Synthesis

1. What was the nature of Newton's synthesis?

4.7 Newton and Hypotheses

1. If Newton could not test the gravitational attraction of every body in the Universe, how could he dare to formulate a "universal" law of gravitation?

2. What is meant by "action at a distance"?
3. What was the popular type of explanation for "action at a distance"? Why did Newton not use this type of explanation?
4. What are two main types of hypotheses used in science?
5. Newton's claim to "frame no hypotheses" seems to refer to hypotheses that cannot be tested. Which of the following claims are not directly testable?
   (a) Plants need sunlight to grow, even on other planets.
   (b) This bandage is guaranteed to be free from germs unless the package is opened.
   (c) Virtual particles exist for a time that is too short for them to affect anything.
   (d) Life exists in the distant galaxies.
   (e) The Earth really does not move, since you would feel the motion if it did.
   (f) Universal gravitation holds between every pair of objects in the Universe.

4.8 The Magnitude of the Gravitational Force

1. How did Newton define the value of R in the force equation?
2. Two large balls of equal size and mass are touching each other. The radius of each is 1 m. In finding their mutual gravitational attraction, what is the value of R that goes into Newton's equation?
3. Two objects attract each other by gravitation. If the mass of each object is doubled, how does the force between them change?
4. Two people attract each other by gravitation. If the distance between them is doubled, but their masses stay the same, how does the force between them change?
5. Finally, if both masses and the distance between two people are doubled, how does the gravitational force between them change?
6. Can a theory ever be absolutely confirmed or proved? If not, does this throw doubt on the usefulness of a theory? Explain.
7. According to Newton's law of action and reaction, the Earth should experience a force and accelerate toward a falling stone.
   (a) How does the force on the Earth compare with the force on the stone?
   (b) How does the Earth's acceleration compare with the stone's acceleration?

4.9 The Value of *G*, and Some Consequences

1. Which of the quantities in the equation $F_{grav} = Gm_1m_2/R^2$ did Cavendish measure?
2. Knowing a value for G, what other information can be used to find the acceleration of gravity, g?
3. The mass of the Sun is about 1000 times the mass of Jupiter. How does the Sun's acceleration, owing to Jupiter's attraction, compare with Jupiter's acceleration owing to the Sun's attraction?
4. Two young persons you know are standing about 1 ft apart. Stating what assumption you make (e.g., about their respective masses, etc.), calculate the approximate value of the attraction between them.

5. What is the difference between a theory and a law in science? Explain with some examples.

4.10 Further Successes

1. How does the Moon cause the water level to rise on both sides of the Earth?
2. Explain why at a point on the ocean shore two high tides and two low tides are observed each day.
3. In which of the following does the Moon produce tides?
   (a) the seas;
   (b) the atmosphere;
   (c) the solid Earth.
4. Why is the calculation of the Moon's motion so difficult?
5. How are the orbits of comets different from the orbits of the planets?
6. Do these differences affect the validity of Newton's law of universal gravitation as applied to comets?

4.11 The Nature of Newton's Work

1. How did the orbit of Mercury challenge Newton's theory?
2. The theories of relativity and quantum mechanics contradict Newtonian physics in some situations. Does this mean that Newtonian physics must now be rejected? If not, why not?
3. In what ways are relativity theory and quantum mechanics "linked" to Newtonian physics?

DISCOVERY QUESTIONS

1. Write a brief outline of the steps Newton took in arriving at the law of universal gravitation. Include the data, assumptions, empirical laws, and hypotheses he used in constructing his theory.
2. Think of one of the theories you have encountered so far in this book. How did the proposer of the theory obtain it? What were the reasons for its acceptance and/or rejection?
3. An apple is attracted to the Earth by the force of gravitation. What is the reaction force to this action? What happens to the Earth as a result? Do we see this reaction? Why or why not?
4. How would you answer the following question: What keeps the Moon up?
5. Since Newton can be regarded as the "culmination of the Scientific Revolution," how would you answer the following:
   (a) What happened to Plato's problem? Was it solved?
   (b) Why do we believe today in the heliocentric theory? What is the evidence for it?
   (c) Is there any fundamental difference between the timeless heavens and the constantly changing Earth?

6. Is Newton's work only of historical interest, or is it still useful today? Explain.
7. What are some of the major consequences of Newton's work on scientists' views of the world?
8. What is the relationship, if any, between physics and our culture and society?
9. Set up a discussion or debate between a person living in the Aristotelian world view before Newton and a person living in the Newtonian world view.

Quantitative

1. Using the value for the Sun's mass $m_{sun} = 1.98 \times 10^{30}$ kg and the mass of the Earth from the text, find the gravitational attraction of the Sun on the Earth.
2. How could you find the attractive force of the Sun on the Earth if you did not know the mass of the Sun?
3. Using the values for the mass and radius of the Earth in the text, find the acceleration of gravity at the Earth's surface in meters per seconds squared. Assume that the Earth is a homogeneous sphere.
4. One of Jupiter's moons, Europa, has an ocean on its surface whose surface is frozen. Some scientists speculate that some form of life may exist below the surface. Find from tables the radius and period of Europa's orbit around Jupiter. From this information, find the mass of Jupiter. How does the mass of Jupiter, the largest planet, compare with the mass of Earth?
5. How high could an astronaut throw a ball on the Moon, if he can throw it to the height of 30 m on Earth?
6. (a) The gravitational force from Earth extends all the way to the Moon and beyond. How far out into space does it extend? Does the gravitational force ever decline to zero?
   (b) Black holes are thought to contain mass that is concentrated into a space approaching a radius of 0. What would happen to the gravitational force if the distance R between the centers of two masses were 0? Can this ever occur?
   (c) Draw a graph of the gravitational force as a function of R; place F on the y-axis and R on the x-axis. Let Gm_1m_2 remain constant.