CHAPTER 9

Einstein and Relativity Theory

9.1 THE NEW PHYSICS

Following Newton's triumph, work expanded not only in mechanics but also in the other branches of physics, in particular, in electricity and magnetism. This work culminated in the late nineteenth century in a new and successful theory of electricity and magnetism based upon the idea of electric and magnetic fields. The Scottish scientist James Clerk Maxwell, who formulated the new electromagnetic field theory, showed that what we observe as light can be understood as an electromagnetic wave. Newton's physics and Maxwell's theory account, to this day, for almost everything we observe in the everyday physical world around us. The motions of planets, cars, and projectiles, light and radio waves, colors, electric and magnetic effects, and currents all fit within the physics of Newton, Maxwell, and their contemporaries. In addition, their work made possible the many wonders of the new electric age that have spread throughout much of the world since the late nineteenth century. No wonder that by 1900 some distinguished physicists believed that physics was nearly complete, needing only a few minor adjustments. No wonder they were so astonished when, just 5 years later, an unknown Swiss patent clerk, who had graduated from the Swiss Polytechnic Institute in Zurich in 1900, presented five major research papers that touched off a major transformation in physics that is still in progress. Two of these papers provided the long-sought definitive evidence for the existence of atoms and molecules; another initiated the development of the quantum theory of light; and the fourth and fifth papers introduced the theory of relativity. The young man's name was Albert Einstein, and this chapter introduces his theory of relativity and some of its many consequences.

Although relativity theory represented a break with the past, it was a gentle break. As Einstein himself put it:

> We have here no revolutionary act but the natural continuation of a line that can be traced through centuries. The abandonment of certain notions connected with space, time, and motion hitherto treated as fundamentals must not be regarded as arbitrary, but only as conditioned by the observed facts.*

The "classical physics" of Newton and Maxwell is still intact today for events in the everyday world on the human scale—which is what we would expect, since physics was derived from and designed for the everyday world. However, when we get away from the everyday world, we need to use relativity theory (for speeds close to the speed of light and for extremely high densities of matter, such as those found in neutron stars and black holes) or quantum theory (for events on the scale of atoms), or the combination of both sets of conditions (e.g., for high-speed events on the atomic scale). What makes these new theories so astounding, and initially difficult to grasp, is that our most familiar ideas and assumptions about such basic concepts as space, time, mass, and causality must be revised in unfamiliar, yet still understandable, ways. But such changes are part of the excitement of science—and it is even more exciting when we realize that much remains to be understood at the frontier of physics. A new world view is slowly emerging to replace the mechanical world view, but when it is fully revealed

---

* *Ideas and Opinions*, p. 246.

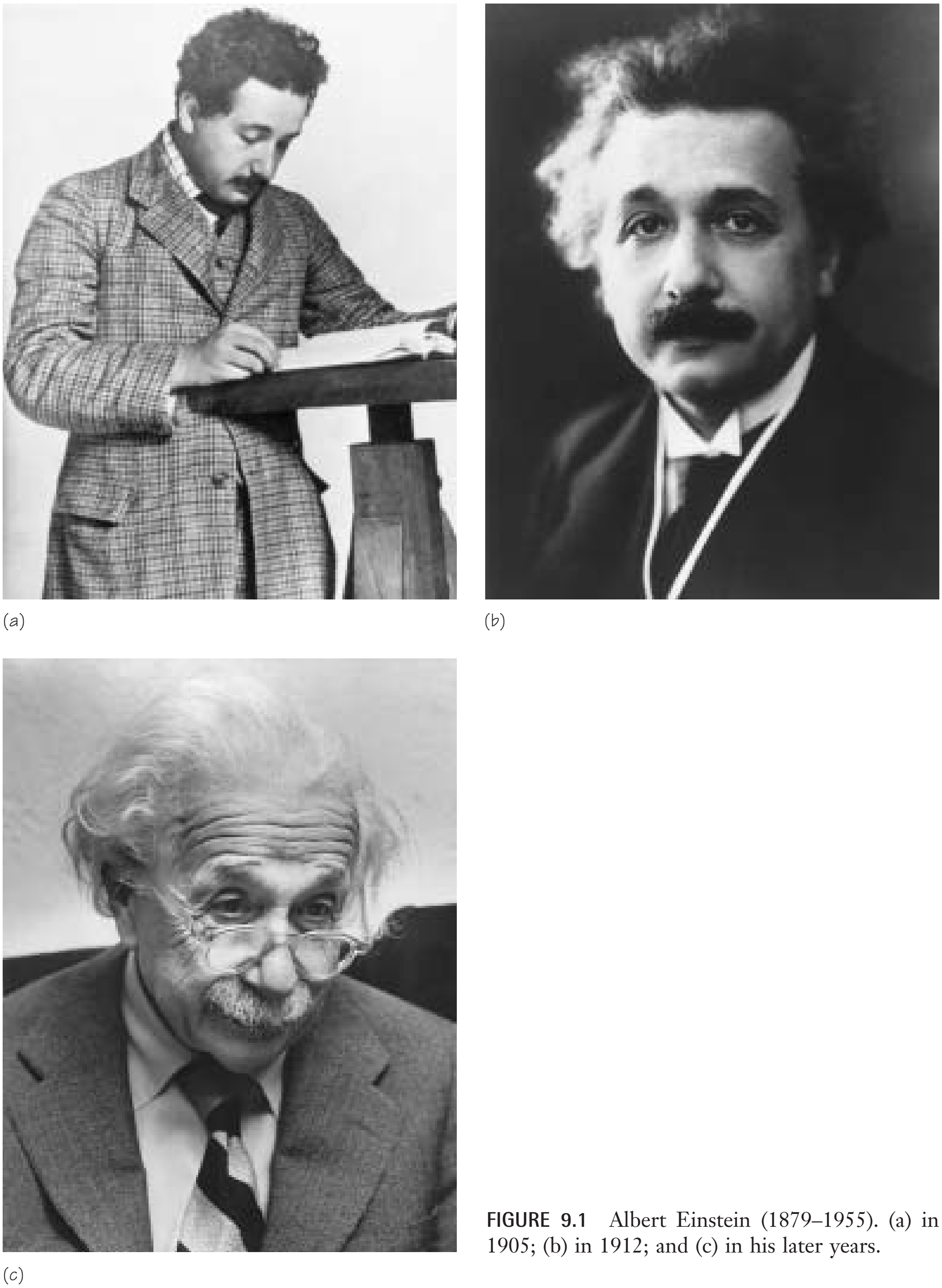


FIGURE 9.1 Albert Einstein (1879–1955). (a) in 1905; (b) in 1912; and (c) in his later years.

it will probably entail some very profound and unfamiliar ideas about nature and our place in it.

9.2 ALBERT EINSTEIN

Obviously to have founded relativity theory and to put forth a quantum theory of light, all within a few months, Einstein had to be both a brilliant physicist and a totally unhindered, free thinker. His brilliance shines throughout his work, his free thinking shines throughout his life.

Born on March 14, 1879, of nonreligious Jewish parents in the southern German town of Ulm, Albert was taken by his family to Munich 1 year later. Albert's father and an uncle, both working in the then new profession of electrical engineering, opened a manufacturing firm for electrical and plumbing apparatus in the Bavarian capital. The firm did quite well in the expanding market for recently developed electrical devices, such as telephones and generators, some manufactured under the uncle's own patent. The Munich business failed, however, after the Einsteins lost a municipal contract to wire a Munich suburb for electric lighting (perhaps similar in our day to wiring fiber-optic cable for TV and high-speed Internet access). In 1894 the family pulled up stakes and moved to Milan, in northern Italy, where business prospects seemed brighter, but they left Albert, then aged 15, behind with relatives to complete his high-school education. The teenager lasted alone in Munich only a half year more. He quit school, which he felt too militaristic, when vacation arrived in December 1894, and headed south to join his family.

Upon arriving in Milan, the confident young man assured his parents that he intended to continue his education. Although underage and without a high-school diploma, Albert prepared on his own to enter the Swiss Federal Polytechnic Institute in Zurich, comparable to the Massachusetts Institute of Technology or the California Institute of Technology, by taking an entrance examination. Deficiencies in history and foreign language doomed his examination performance, but he did well in mathematics and science, and he was advised to complete his high-school education, which would ensure his admission to the Swiss Polytechnic. This resulted in his fortunate placement for a year in a Swiss high school in a nearby town. Boarding in the stimulating home of one of his teachers, the new pupil blossomed in every respect within the free environment of Swiss education and democracy.

Einstein earned high marks, graduated in 1896, and entered the teacher training program at the Swiss Polytechnic, heading for certification as a high-school mathematics and physics teacher. He was a good but not an outstanding student, often carried along by his friends. The mathematics and physics taught there were at a high level, but Albert greatly disliked the lack of training in any of the latest advances in Newtonian physics or Maxwellian electromagnetism. Einstein mastered these subjects entirely by studying on his own.

One of Einstein's fellow students was Mileva Marić, a young Serbian woman who had come to Zurich to study physics, since at that time most other European universities did not allow women to register as full-time students. A romance blossomed between Mileva and Albert. Despite the opposition of Einstein's family, the romance flourished. However, Mileva gave birth to an illegitimate daughter in 1902. The daughter, Liserl, was apparently given up for adoption. Not until later did Einstein's family finally accede to their marriage, which took place in early 1903. Mileva and Albert later had two sons, Hans Albert and Eduard, and for many years were happy together. But they divorced in 1919.

Another difficulty involved Einstein's career. In 1900 and for sometime after, it was headed nowhere. For reasons that are still unclear, probably anti-Semitism and personality conflicts, Albert was continually passed over for academic jobs. For several years he lived a discouraging existence of temporary teaching positions and freelance tutoring. Lacking an academic sponsor, his doctoral dissertation which provided further evidence for the existence of atoms was not accepted until 1905. Prompted by friends of the family, in 1902 the Federal Patent Office in Bern, Switzerland, finally offered Einstein a job as an entry-level patent examiner. Despite the full-time work, 6 days per week, Albert still found time for fundamental research in physics, publishing his five fundamental papers in 1905.

The rest, as they say, was history. As the importance of his work became known, recognized at first slowly, Einstein climbed the academic ladder, arriving at the top of the physics profession in 1914 as Professor of Theoretical Physics in Berlin.

In 1916, Einstein published his theory of general relativity. In it he provided a new theory of gravitation that included Newton's theory as a special case. Experimental confirmation of this theory in 1919 brought Einstein world fame. His earlier theory of 1905 is now called the theory of special relativity, since it excluded accelerations.

When the Nazis came to power in Germany in January 1933, Hitler being appointed chancellor, Einstein was at that time visiting the United States, and vowed not to return to Germany. He became a member of the newly formed Institute for Advanced Study in Princeton. He spent the rest of his life seeking a unified theory which would include gravitation and electromagnetism. As World War II was looming, Einstein signed a letter to President Roosevelt, warning that it might be possible to make an "atomic bomb," for which the Germans had the necessary knowledge. (It was later found that they had a head-start on such research, but failed.) After World War II, Einstein devoted much of his time to organizations advocating world agreements to end the threat of nuclear warfare. He spoke and acted in favor of the founding of Israel. His obstinate search to the end for a unified field theory was unsuccessful; but that program, in more modern guise, is still one of the great frontier activities in physics today. Albert Einstein died in Princeton on April 18, 1955.

9.3 THE RELATIVITY PRINCIPLE

Compared with other theories discussed so far in this book, Einstein's theory of relativity is more like Copernicus's heliocentric theory than Newton's universal gravitation. Newton's theory is what Einstein called a "constructive theory." It was built up largely from results of experimental evidence (Kepler, Galileo) using reasoning, hypotheses closely related to empirical laws, and mathematical connections. On the other hand, Copernicus' theory was not based on any new experimental evidence but primarily on aesthetic concerns. Einstein called this a "principle theory," since it was based on certain assumed principles about nature, of which the deduction could then be tested against the observed behavior of the real world. For Copernicus these principles included the ideas that nature should be simple, harmonious, and "beautiful." Einstein was motivated by similar concerns. As one of his closest students later wrote,

> You could see that Einstein was motivated not by logic in the narrow sense of the word but by a sense of beauty. He was always looking for beauty in his work. Equally he was moved by a profound religious sense fulfilled in finding wonderful laws, simple laws in the Universe.*

Einstein's work on relativity comprises two parts: a "special theory" and a "general theory." The special theory refers to motions of observers and events that do not exhibit any accelerations. The velocities remain uniform. The general theory, on the other hand, does admit accelerations.

Einstein's special theory of relativity began with aesthetic concerns which led him to formulate two fundamental principles about nature. Allowing

---

* Banesh Hoffmann in *Strangeness in the Proportion*, H. Woolf, ed., see Further Reading.

himself to be led wherever the logic of these two principles took him, he then derived from them a new theory of the basic notions of space, time, and mass that are at the foundation of all of physics. He was not constructing a new theory to accommodate new or puzzling data, but deriving by deduction the consequences about the fundamentals of all physical theories from his basic principles.

Although some experimental evidence was mounting against the classical physics of Newton, Maxwell, and their contemporaries, Einstein was concerned instead from a young age by the inconsistent way in which Maxwell's theory was being used to handle relative motion. This led to the first of Einstein's two basic postulates: the Principle of Relativity, and to the title of his relativity paper, "On the Electrodynamics of Moving Bodies."

Relative Motion

But let's begin at the beginning: *What is relative motion?* As you saw in Chapter 1, one way to discuss the motion of an object is to determine its average speed, which is defined as the distance traveled during an elapsed time, say, 13.0 cm in 0.10 s, or 130 cm/s. In Chapter 1 a small cart moved with that average speed on a tabletop, and the distance traveled was measured relative to a fixed meter stick. But suppose the table on which the meter stick rests and the cart moves is itself rolling forward in the same direction as the cart, at 100 cm/s relative to the floor. Then *relative to a meter stick on the floor*, the cart is moving at a different speed, 230 cm/s (100 + 130), while the cart is still moving at 130 cm/s *relative to the tabletop*. So, in measuring the average speed of the cart, we have first to specify what we will use as our reference against which to measure the speed. Is it the tabletop, or the floor, or something else? The reference we finally decide upon is called the "reference frame" (since we can regard it to be as a picture frame around the observed events). *All speeds are thus defined relative to the reference frame we choose.*

But notice that if we use the floor as our reference frame, it is not at rest either. It is moving relative to the center of the Earth, since the Earth is

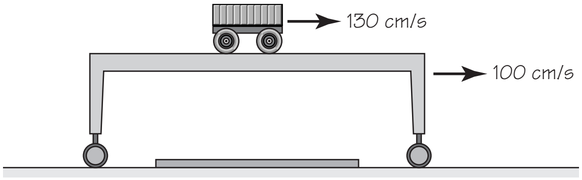


FIGURE 9.2 Moving cart on a moving table.

rotating. Also, the center of the Earth is moving relative to the Sun; and the Sun is moving relative to the center of the Milky Way galaxy, and on and on. . . . Do we ever reach an end? Is there something that is at *absolute rest*? Newton and almost everyone after him until Einstein thought so. For them, it was space itself that was at absolute rest. In Maxwell's theory this space is thought to be filled with a substance that is not like normal matter. It is a substance, called the "ether," that physicists for centuries hypothesized to be the carrier of the gravitational force. For Maxwell, the ether itself is at rest in space, and accounts for the behavior of the electric and magnetic forces and for the propagation of electromagnetic waves (further details in Chapter 12).

Although every experimental effort during the late nineteenth century to detect the resting ether had ended in failure, Einstein was most concerned from the start, not with this failure, but with an inconsistency in the way Maxwell's theory treated relative motion. Einstein centered on the fact that it is only the relative motions of objects and observers, rather than any supposed absolute motion, that is most important in this or any theory. For example, in Maxwell's theory, when a magnet is moved at a speed v relative to a fixed coil of wire, a current is induced in the coil, which can be calculated ahead of time by a certain formula (this effect is further discussed in Chapter 11). Now if the magnet is held fixed and the coil is moved at the same speed v, the same current is induced but a *different* equation is needed to calculate it in advance. Why should this be so, Einstein wondered, since only the relative speed v counts? Since absolutes of velocity, as of space and time, neither appeared in real calculations nor could be determined experimentally, Einstein declared that the absolutes, and on their basis in the supposed existence of the ether, were "superfluous," unnecessary. The ether seemed helpful for imagining how light waves traveled—but it was not needed. And since it could not be detected either, after Einstein's publication of his theory most physicists eventually came to agree that it simply did not exist. For the same reason, one could dispense with the notions of absolute rest and absolute motion. In other words, Einstein concluded, *all motion, whether of objects or light beams, is relative motion*. It must be defined relative to a specific reference frame, which itself may or may not be in motion relative to another reference frame.

The Relativity Principle—Galileo's Version

You saw in Section 3.10 that Galileo's thought experiments on falling objects dropped from moving towers and masts of moving ships, or butterflies trapped inside a ship's cabin, indicated that to a person within a reference frame, whether at rest or in uniform relative motion, there is no

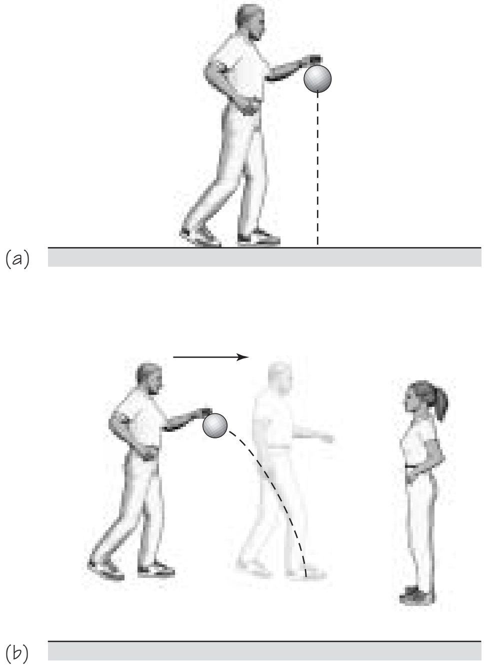


FIGURE 9.3 (a) Falling ball as seen by you as you walk forward at constant speed; (b) falling ball as seen by stationary observer.

way for that person to find out the speed of his *own* reference frame from any mechanical experiment done *within* that frame. Everything happens within that frame as if the frame is at rest.

But how does it look to someone outside the reference frame? For instance, suppose you drop a ball in a moving frame. To you, riding with the moving frame, it appears to fall straight down to the floor, much like a ball dropped from the mast of a moving ship. But what does the motion of the ball look like to someone who is not moving with you, say a classmate standing on the shore as your ship passes by? Or sitting in a chair and watching you letting a ball drop as you are walking by? Try it!

Looking at this closely, your classmate will notice that from her point of view the ball does not fall straight down. Rather, as with Galileo's falling ball from the mast or the moving tower, the ball follows the parabolic trajectory of a projectile, with uniform velocity in the horizontal direction as well as uniform acceleration in the vertical direction.

The surprising result of this experiment is that two different people in two different reference frames will describe the same event in two different ways. As you were walking or sailing past, you were in a reference frame with respect to which the ball is at rest before being released. When you let it go, you see it falling straight down along beside you, and it lands at your feet. But persons sitting in chairs or standing on the shore, in their own reference frame, will report that they see something entirely different: a ball that starts out with you—not at rest but in forward motion—and on release it moves—not straight down, but on a parabola toward the ground, hitting the ground at your feet. Moreover, this is just what they would expect to see, since the ball started out moving horizontally and then traced out the curving path of a projectile.

So who is correct? Did the ball fall straight down or did it follow the curving path of a projectile? Galileo's answer was: *both are correct*. But how can that be? How can there be two different observations and two different explanations for one physical event, a ball falling to someone's feet? The answer is that different observers observe the same event differently when they are observing the event from different reference frames in relative motion. The ball starts out stationary relative to one frame (yours), whereas it is, up to its release, in constant (uniform) motion relative to the other reference frame (your classmate's). Both observers see everything happen as they expect it from Newton's laws applied to their situation. But what they see is different for each observer. Since there is no absolute reference frame (no reference frame in uniform velocity is better or preferred over any other moving with uniform velocity), there is no absolute motion, and their observations made by both observers are equally valid.

Galileo realized that the person who is at rest relative to the ball could not determine by any such mechanical experiment involving falling balls, inclined planes, etc., whether or not he is at rest or in uniform motion relative to anything else, since all of these experiments will occur as if he is simply at rest. A ball dropping from a tower on the moving Earth will hit the base of the tower as if the Earth were at rest. Since we move with the Earth, as long as the Earth can be regarded as moving with uniform velocity (neglecting during the brief period of the experiment that it actually rotates), there is no mechanical experiment that will enable us to determine whether or not we are really at rest or in uniform motion.

Note: The observation of events are frame dependent. But the laws of mechanics are not. They are the same in reference frames that are at rest or in relative uniform motion. All objects that we observe to be moving relative to us will also follow the same mechanical laws (Newton's laws, etc.). As discussed in Section 3.10, this statement applied to mechanical phenomena is known as the *Galilean relativity principle*.

The Relativity Principle—Einstein's Version

In formulating his theory of relativity, Einstein expanded Galileo's principle into the *Principle of Relativity* by including *all of the laws of physics*, such as the laws governing light and other effects of electromagnetism, not just mechanics. Einstein used this principle as one of the two postulates of his theory of relativity, from which he then derived the consequences by deduction. Einstein's *Principle of Relativity* states:

> All the laws of physics are exactly the same for every observer in every reference frame that is at rest or moving with uniform relative velocity. This means that there is no experiment that they can perform in their reference frames that would reveal whether or not they are at rest or moving at uniform velocity.

Reference frames that are at rest or in uniform velocity relative to another reference frame have a technical name. They are called *inertial reference frames* (since Newton's law of inertia holds in them). Reference frames that are accelerating relative to each other are called *noninertial reference frames*. They are *not* included in this part of the theory of relativity. That is why this part of the theory of relativity is called the *theory of special relativity*. It is restricted to inertial reference frames, those which are either at rest or moving with uniform velocity relative to each other.

Notice that, according to Einstein's Relativity Principle, Newton's laws of motion and all of the other laws of physics remain the same for phenomena occurring in any of the inertial reference frames. This principle does *not* say that "everything is relative." On the contrary, it asks you to look for relationships that do *not* change when you transfer your attention from one moving reference frame to another. The physical measurements but not the physical laws depend on the observer's frame of reference.

9.4 CONSTANCY OF THE SPEED OF LIGHT

The Relativity Principle is one of the two postulates from which Einstein derived the consequences of relativity theory. The other postulate concerns the speed of light, and it is especially important when comparing observations between two inertial reference frames in relative motion, since we rely chiefly on light to make observations.

You recall that when Einstein quit high school at age 15 he studied on his own to be able to enter the Swiss Polytechnic Institute. It was probably during this early period that Einstein had a remarkable insight. He asked himself what would happen if he could move fast enough in space to catch up with a beam of light. Maxwell had shown that light is an electromagnetic wave propagating outward at the speed of light. If Albert could

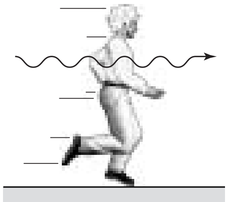


FIGURE 9.4 Running alongside a beam of light.

ride alongside, he would not see a wave propagating. Instead, he would see the "valleys" and "crests" of the wave fixed and stationary with respect to him. This contradicted Maxwell's theory, in which no such "stationary" landscape in free space was possible. From these and other, chiefly theoretical considerations, Einstein concluded by 1905 that Maxwell's theory must be reinterpreted: the speed of light will be exactly the same—a universal constant—for all observers, no matter whether they move (with constant velocity) relative to the source of the light. This highly original insight became Einstein's second postulate of special relativity, the *Principle of the Constancy of the Speed of Light*:

> Light and all other forms of electromagnetic radiation are propagated in empty space with a constant velocity *c* which is independent of the motion of the observer or the emitting body.

Einstein is saying that, whether moving at uniform speed toward or away from the source of light or alongside the emitted light beam, any observer always measures the exact same value for the speed of light in a vacuum, which is about 3.0×10^8 m/s or 300,000 km/s (186,000 mi/s). (More precisely, it is 299,790 km/s.) This speed was given the symbol *c* for "constant." If light travels through glass or air, the speed will be slower, but the speed of light in a vacuum is one of the *universal physical constants of nature*. (Another is the gravitational constant G.) It is important to note that, again, this principle holds only for observers and sources that are in inertial reference frames. This means they are moving at uniform velocity or are at rest relative to each other.

In order to see how odd the principle of the constancy of the speed of light really is, let's consider a so-called "thought experiment," an experiment that one performs only in one's mind. It involves two "virtual student researchers." One, whom we'll call Jane, is on a platform on wheels moving at a uniform speed of 5 m/s toward the second student, John, who is standing on the ground. While Jane is moving, she throws a tennis ball to John at 7 m/s. John catches the ball, but before he does he quickly measures its speed (this is only a thought experiment!). What speed does he obtain? . . . The answer is 5 m/s + 7 m/s = 12 m/s, since the two speeds combine.

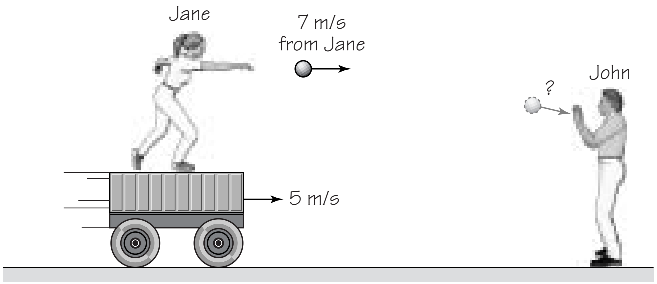


FIGURE 9.5 Ball thrown from a cart moving in the same direction. Jane is moving at 5 m/s, and the ball is thrown to John at a speed of 7 m/s.

Let's try it in the opposite direction. Jane is on the platform now moving at 5 m/s *away* from John. She again tosses the ball to John at 7 m/s, who again measures its speed before catching it. What speed does he measure? . . . This time it's −5 m/s + 7 m/s = 2 m/s. The velocities are subtracted. All this was as expected.

Now let's try these experiments with light beams instead of tennis balls. As Jane moves toward John, she aims the beam from a laser pen at John (being careful to avoid his eyes). John has a light detector that also measures the speed of the light. What is the speed of the light that he measures? . . . Neglecting the minute effect of air on the speed of light, Jane and John are surprised to find that Einstein was right: The speed is exactly the speed of light, no more, no less. They obtain the same speed when the platform moves away from John. In fact, even if they get the speed of the platform almost up to nearly the speed of light itself (possible only in a thought experiment), the measured speed of light is still the same in both instances. Strange as it seems, the speed of light (or of any electromagnetic wave) always has the same value, no matter what the relative speed is of the source and the observer.

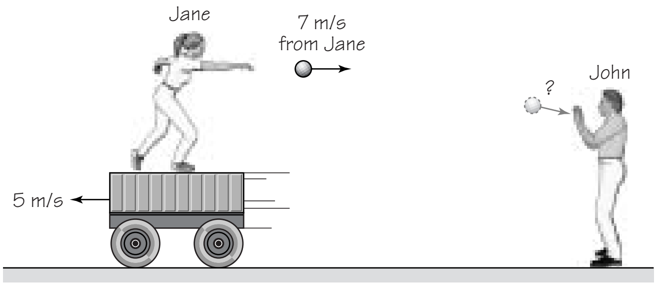


FIGURE 9.6 Ball thrown from a cart moving in the opposite direction.

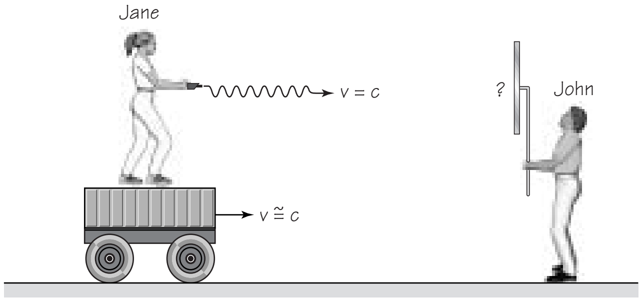


FIGURE 9.7 Light beam directed from a moving cart.

Let's consider some consequences that followed when Einstein put together the two fundamental postulates of special relativity theory, the Principle of Relativity and the Principle of the Constancy of the Speed of Light in space.

9.5 SIMULTANEOUS EVENTS

Applying the two postulates of relativity theory to a situation similar to Galileo's ship, Einstein provided a simple but profound thought experiment that demonstrated a surprising result. He discovered that two events that occur simultaneously for one observer may not occur simultaneously for another observer in relative motion with respect to the events. In other words, the simultaneity of events is a relative concept. (Nevertheless, the laws of physics regarding these events still hold.)

Einstein's thought experiment, an experiment that he performed through logical deduction, is as follows in slightly updated form. An observer, John, is standing next to a perfectly straight level railroad track. He is situated at the midpoint between positions A and B in Figure 9.8. Imagine that he is holding an electrical switch which connects wires of equal length to lights bulbs placed at A and B. Since he is at the midpoint between A and B, if he closes the switch, the bulbs will light up, and very shortly thereafter John will see the light from A and from B arriving at his eyes at the same moment. This is because the light from each bulb, traveling at the constant speed of light and covering the exact same distance to John from each bulb, will take the exact same time to reach his eyes. John concludes from this that the two light bulbs lit up simultaneously.

Now imagine a second observer, Jane, standing at the middle of a flat railroad car traveling along the track at a very high uniform speed to the right. Jane and John have agreed that when she reaches the exact midpoint between A and B, John will instantly throw the switch, turning on the light

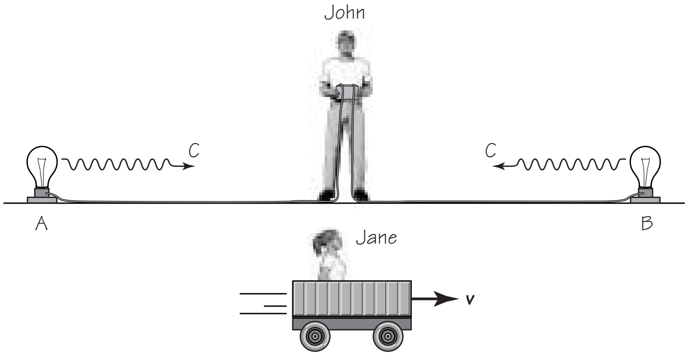


FIGURE 9.8 Einstein's thought experiment demonstrating the relativity of simultaneous events.

bulbs. (Since this is a thought experiment, we may neglect his reaction time, or else he might use a switch activated electronically.)

John and Jane try the experiment. The instant Jane reaches the midpoint position between A and B, the switch is closed, the light bulbs light up, and John sees the flashes simultaneously. But Jane sees something different: to her the flashes do *not* occur simultaneously. In fact, the bulb at B appeared to light up before the bulb at A. Why? Because she is traveling toward B and away from A and, because the speed of light is the same regardless of the motion of the observer, she will encounter the beam from B before the beam from A reaches her. Consequently, she will see the flash at B before she sees the flash at A. The conclusion: The two events that John perceives to occur simultaneously do not occur simultaneously for Jane. The reasons for this discrepancy are that the speed of light is the same for both observers and that each observer is moving in a different way relative to the events in question.

It might be argued that Jane could make a calculation in which she computed her speed and the speed of light, and then very simply find out if the flashes actually occurred as she saw them or as John claimed to see them. However, if she does this, then she is accepting a specific frame of reference: That is, she is assuming that she is the moving observer and that John is the stationary observer. But according to the relativity postulate motions are relative, and she need not assume that she is moving since there is no preferred frame of reference. Therefore she could just as well be the stationary observer, and John, standing next to the track, could be the moving observer! If that is so, then Jane could claim that the flash at B actually did occur before the flash at A and that John perceived them to occur simultaneously only because from her point of view he was moving toward

A and away from B. On the other hand, John could argue just the reverse, that he is at rest and it is Jane who is moving.

Which interpretation is correct? There is no "correct" interpretation because there is no preferred frame of reference. Both observers are moving relative to each other. They can agree on what happened only if they agree on the frame of reference, but that agreement is purely arbitrary.

The conclusion that the simultaneity of two events, such as two flashes from separate light bulbs, depends upon the motion of the observer, led to the possibility that time itself might also be a relative concept when examined in view of the relativity postulates.

9.6 RELATIVITY OF TIME

Let's see what happens to the measurement of time when understood through special relativity.

We'll follow Einstein's original argument and examine another, somewhat updated thought experiment. In this experiment one observer—again we'll call her Jane—is in a spaceship moving at an extremely fast uniform speed relative to the Earth and in the horizontal direction relative to another observer, John, who is stationary on the Earth. In Jane's spaceship (i.e., in her reference frame) there is a clock that measures time in precise intervals by using a laser pulse. The pulse travels straight up from a laser, hits a mirror, and is reflected back down. When the pulse returns to the starting point, it is detected by a photosensor, which then registers the elapsed time Δt, a fraction of a second, say, 10^{-7} s, and emits another pulse upward. Since the speed of light is constant and the distance that it travels is fixed, it takes the second pulse the exact same amount of time to make the round trip. So another 10^{-7} s is registered by the detector. These identical time intervals are used as a clock to keep time.

Since Jane is traveling at uniform velocity, Einstein's Principle of Rela-

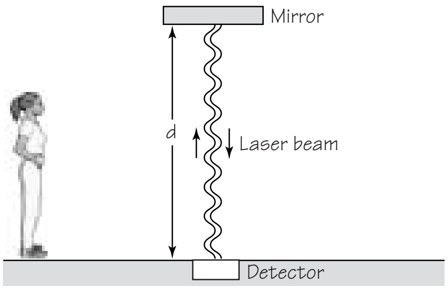


FIGURE 9.9 Laser clock in spaceship (as seen from spaceship frame of reference).

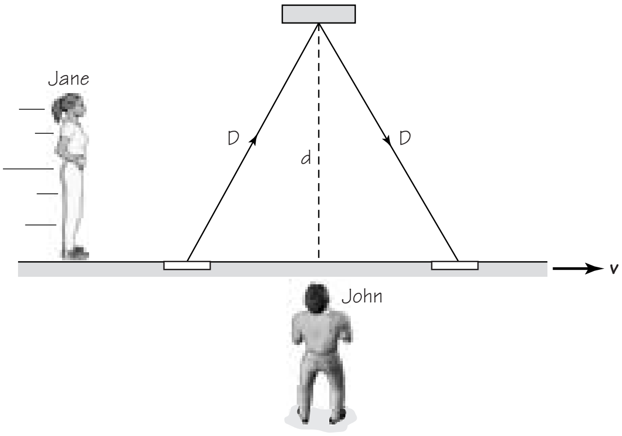


FIGURE 9.10 Laser clock in spaceship (as seen from an outside observer's frame of reference).

tivity tells her that the clock behaves exactly as it would if she were at rest. In fact, according this principle, she could not tell from this experiment (or any other) whether her ship is at rest or moving relative to John, without looking outside the spaceship. But to John, who is not in her reference frame but in his own, she appears to him to be moving forward rapidly in the horizontal direction relative to him. (Of course, it might be John who is moving backward, while Jane is stationary; but the observation and the argument that follows will be the same.)

Observing Jane's laser clock as her spaceship flies past him, what does John see? Just as before, in the experiment with the ball observed to be falling toward the floor when released by a moving person, John sees something quite different from what Jane sees. Because her spaceship is moving with respect to him, he observes that the light pulse follows a diagonal path upward to the upper mirror and another diagonal path downward to the detector. Let us give the symbol t' for the time he measures for the round trip of the light pulse.

Here enters the second postulate: the measured speed of light must be the same as observed by both John and Jane. But the distance the light pulse travels during one round trip, as Jane sees it, is shorter than what John sees. Call the total distance the pulse travels from the emitter to the upper mirror and back d for Jane and d' for John. The speed of light, c, which is the same for each, is

$$\text{Jane:} \qquad c = \frac{d}{t},$$

$$\text{John:} \qquad c = \frac{d'}{t'}.$$

DERIVATION OF TIME DILATION: THE LIGHT CLOCK

The “clock” consists of a stick of length *l* with a mirror and a photodetector *P* at each end. A flash of light at one end is reflected by the mirror at the other end and returns to the photodetector next to the light source. Each time a light flash is detected, the clock “ticks” and emits another flash.

Diagram (a) below shows the clock as seen by an observer riding with the clock. The observer records the time *t* between ticks of the clock. For this observer, the total distance traveled by the light pulse during the time *t* is $d = 2l$. Since the light flash travels at the speed of light *c*:

$$d = 2l = ct.$$

So

$$l = ct/2.$$

Diagram (b) shows the same clock as seen by an observer who is “stationary” in his or her own framework, with the clock apparatus moving by. This observer observes and records the time t' between ticks of the clock. For this observer, the total distance traveled by the light beam is d' in time t'. Since light travels at the same speed for all observers moving at uniform speed relative to each other, we have

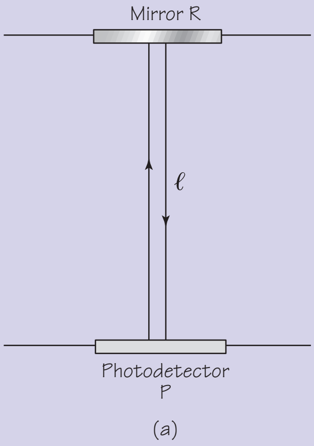
$$d' = ct'.$$

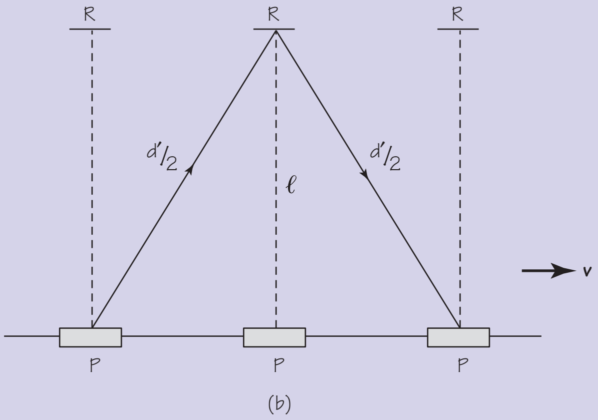
Let’s look at the left side of drawing (b). Here the motion of the clock, the vertical distance *l*, and the motion of the light beam form a right triangle. The base of the triangle is the distance traveled by the clock in time $t'/2$, which is $vt'/2$. The distance the beam travels in reaching the mirror is $d'/2$. Using the Pythagorean theorem, we obtain

$$\left(\frac{d'}{2}\right)^2 = l^2 + \left(\frac{vt'}{2}\right)^2.$$

From the above, we can substitute $d' = ct'$ and $l = ct/2$:

$$\left(\frac{ct'}{2}\right)^2 = \left(\frac{ct}{2}\right)^2 + \left(\frac{vt'}{2}\right)^2.$$





Squaring and canceling like terms, we have

$$c^2t'^2 = c^2t^2 + v^2t'^2.$$

Now, let's solve for t':

$$c^2t'^2 - v^2t'^2 = c^2t^2,$$

$$t'^2\,(c^2 - v^2) = c^2t^2,$$

$$t'^2 = \frac{c^2t^2}{c^2 - v^2},$$

$$t'^2 = \frac{t^2}{1 - v^2/c^2},$$

or

$$t' = \frac{t}{\sqrt{1 - v^2/c^2}},$$

Since $1 - v^2/c^2$ is here always less than 1, the denominator is less than 1, and the fraction is *larger* than t alone. Thus, the time interval t' registered by the clock as seen by the stationary observer is "dilated" compared to the time interval t registered by the clock as seen by the observer riding with the clock. In other words, the moving clock appears to run slower as measured by the stationary observer than when the clock is not moving with respect to the observer. Note also the crucial role of Einstein's second postulate in this derivation.

Since d' is larger than d, t' must be larger than t, in order for the ratios on the right side of both equations to have the same value, c. This means that the time interval ($\Delta t'$) for the round trip of the light pulse, as registered on the clock as John observes it, is longer than the time interval (Δt) registered on the clock as Jane observes it.

The surprising conclusion of this thought experiment (which is really a deduction from the postulates of relativity theory) is:

> Time intervals are not absolute and unchanging, but relative. A clock (such as Jane's), or any repetitive phenomenon which is moving relative to a stationary observer appears to the stationary observer to run slower than it appears to do when measured by the observer moving *with* the clock—and it appears to run slower the faster the clock is moving. This is known as *time dilation.*

Just how much slower does a clock seem when it is moving past an observer? To get the answer, you can use the diagram in Figure 9.10 of John and Jane and apply the Pythagorean theorem. After a bit of basic algebra (see the derivation in the insert), you obtain the exact relationship between the time elapsed interval registered by a clock that is stationary with respect to the observer (as in the case of Jane)—call it now ΔT_s—and the time elapsed interval for the same phenomenon—call it ΔT_{m}—as measured by someone who observes the clock in motion at constant velocity v (as in the case of John). The result is given by the following equation:

$$\Delta T_m = \frac{\Delta T_{\mathrm{s}}}{\sqrt{1 - \dfrac{v^2}{c^2}}}.$$

In words: ΔT_{m}, John's observation of time elapsed registered by the moving clock, is different from ΔT_{s}, Jane's observation of time elapsed registered on the same clock, which is stationary in her frame, by the effect of the factor $\sqrt{1 - v^2/c^2}$ in the denominator.

9.7 TIME DILATION

What may make the equation for time dilation appear complicated is the term in the square root, which contains much of the physics. Study this equation and all of the symbols in it. The symbol c is the speed of light, and v is the speed of the clock moving relative to the observer measuring the time elapsed interval ΔT_{m}. As shown on page 427, for actual objects v is always less than c. Therefore v/c is always less than one, and so is v^2/c^2. In the equation on this page, v^2/c^2 is subtracted from 1, and then you take the square root of the result and divide it into ΔT_{s}, the time elapsed interval registered by the "stationary" clock.

Before we look at the full meaning of what the equation tries to tell us, consider a case where $v = 0$, for example, when Jane's spaceship has stopped relative to the Earth where John is located. If $v = 0$, then v^2/c^2 will be zero, so $1 - v^2/c^2$ is just 1. The square root of 1 is also 1; so our equation reduces to $\Delta T_{\mathrm{m}} = \Delta T_{\mathrm{s}}$: The time interval seen by John is the same as seen by Jane, when both are at rest with respect to each other, as we of course expect.

Now if v is not zero but has some value up to but less than c, then v^2/c^2 is a decimal fraction; so $1 - v^2/c^2$ and its square root are also decimal fractions, less than 1. (Confirm this by letting v be some value, say $\frac{1}{2}c$.) Dividing a decimal fraction into ΔT_{s} will result in a number larger than ΔT_{s}; so by our equation giving ΔT_{m}, ΔT_{m} will turn out to be larger than ΔT_{s}. In other words, the time interval as observed by the stationary observer watching the moving clock is larger (longer) than it would be for someone who is riding with the clock. The clock appears to the observer to run slower.

What Happens at Very High Speed?

Let's see what happens when the speed of the moving clock (or any repetitive process) is extremely fast, say 260,000 km/s (161,000 mi/s) relative to another inertial reference frame. The speed of light c in vacuum is, as always, about 300,000 km/s. When the moving clock registers a time interval of 1 s in its own inertial frame ($\Delta T_{\mathrm{s}} = 1$ s), what is the time interval for someone who watches the clock moving past at the speed of 260,000 km/s? To answer this, knowing that ΔT_{s} is 1 s, we can find ΔT_{m} by substituting the relevant terms into the equation for ΔT_{m}:

$$\left[\frac{v}{c}\right]^2 = \left[\frac{260{,}000\ \mathrm{km/s}}{300{,}000\ \mathrm{km/s}}\right]^2$$

$$= [0.867]^2 = 0.75.$$

Therefore

$$1 - \sqrt{\frac{v^2}{c^2}} = \sqrt{1 - 0.75}$$

$$= \sqrt{0.25} = 0.5.$$

So

$$\Delta T_{\mathrm{m}} = \frac{1\ \mathrm{s}}{0.5}$$

$$= 2\ \mathrm{s}.$$

This result says that a clock moving at 260,000 km/s that registers an interval of 1 s in its own inertial frame appears to an observer at rest relative to the clock to be greatly slowed down. While the person riding with the clock registers a 1-s interval, the resting observer will measure it (with respect to his own clock) to be 2 s. Note again that the clock does not seem to be slowed down at all to the person moving with the clock; but to the outside observer in this case the time interval has "dilated" to exactly double the amount.

What Happens at an Everyday Speed?

Notice also in the previous situation that we obtain a time dilation effect of as little as two times only when the relative speed is 260,000 km/s, which is nearly 87% of the speed of light. For slower speeds, the effect decreases very rapidly, until at everyday speeds we cannot notice it at all, except in very delicate experiments. For example, let's look at a real-life situation, say a clock ticking out a 1-s interval inside a jet plane, flying at the speed of sound of 760 mi/hr, which is about 0.331 km/s. What is the corresponding time interval observed by a person at rest on the ground? Again we substitute into the expression for time dilation.

$$\left[\frac{v}{c}\right]^2 = \left[\frac{0.331\ \text{km/s}}{300{,}000\ \text{km/s}}\right]^2$$

$$= [1.10 \times 10^{-6}]^2 = 1.22 \times 10^{-12}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - (1.22 \times 10^{-12})}$$

$$= \sqrt{0.99999999999878} = 0.99999999999938.$$

So

$$\Delta T_{\text{m}} = \frac{1\ \text{s}}{0.99999999999938}$$

$$= 1.00000000000061\ \text{s}.$$

With such an incredibly minute amount of time dilation, no wonder this effect was never observed earlier! Because the effect is so tiny, Newton's physics is still fine for the everyday world of normal speeds for which it was designed. This is also why it is false to say (as Einstein never did) that relativity theory proved Newton wrong. Nevertheless, the effect on moving clocks is there, and was in fact confirmed in a famous experiment involving a very precise atomic clock flown around the world on a jet airliner. It has also been tested and confirmed by atomic clocks flown on satellites and on the space shuttle at speeds of about 18,000 mi/hr. But the effect is so small that it can be neglected in most situations. It becomes significant only at relative speeds near the speed of light—which is the case in high-energy laboratory experiments and in some astrophysical phenomena.

What Happens When the Speed Reaches the Speed of Light?

If we were to increase the speed of an object far beyond 260,000 km/s, the time dilation effect becomes more and more obvious, until, finally, we approach the speed of light $v = c$. What happens as this occurs? Examining the time dilation equation, v^2/c^2 would approach 1 as v approaches c, so the denominator in the equation, $\Delta T_m = \Delta T_s/\sqrt{(1 - v^2/c^2)}$, would become smaller and smaller, becoming zero at $v = c$. As the denominator approaches zero, the fraction $\Delta T_s/\sqrt{(1 - v^2/c^2)}$ would grow larger and larger without limit, approaching infinity at $v = c$. And ΔT_m would thus become infinite when the speed reaches the speed of light c. In other words, a time interval of 1 s (or any other amount) in one system would be, by measurement with the clock in the other system, an infinity of time; the moving clock will appear to have stopped!

What Happens If v Should Somehow Become Greater Than c?

If this could happen, then v^2/c^2 would be greater than 1, so $(1 - v^2/c^2)$ would be negative. What is the square root of a negative number? You will recall from mathematics that there is no number that, when squared, gives a negative result. So the square root of a negative number itself has no physical reality. It is often called an "imaginary number." In practice, this means that objects cannot have speeds greater than c. This is one reason that the speed of light is often regarded as the "speed limit" of the Universe. *Neither objects nor information can travel faster in vacuum than does light.* As you will see in Section 9.9, nothing that has mass can even reach the speed of light, since c acts as an asymptotic limit of the speed.

Is It Possible to Make Time Go Backward?

The only way for this to happen would be if the ratio $\Delta T_s/\sqrt{(1 - v^2/c^2)}$ is negative, indicating that the final time after an interval has passed is less than the initial time. As you will also recall from mathematics, the solution of every square root has two values, one positive and one negative. Usually in physics we can ignore the negative value because it has no physical meaning. But if we choose it instead, we would obtain a negative result, suggesting that time, at least in theory, would go backward. But this would also mean that mass and energy are negative. That could not apply to ordinary matter, which obviously has positive mass and energy.

In Sum

You will see in the following sections that the square root in the equation for time dilation also appears in the equations for the relativity of length and mass. So it is important to know its properties at the different values of the relative speed. Because it is so important in these equations, the square root $\sqrt{(1 - v^2/c^2)}$ is often given the symbol γ, the Greek letter gamma.

We summarize the properties of $\gamma \equiv \sqrt{1 - v^2/c^2}$, discussed in this section:

$v = 0$,	$\gamma = 1$,
$0 < v < c$,	$\gamma =$ a fraction between 0 and 1, depending on the value of v^2/c^2
$v = 260{,}000$ km/s,	$\gamma = 0.5$,
$v = c$,	$\gamma = 0$,
$v > c$,	$\gamma =$ imaginary.

9.8 RELATIVITY OF LENGTH

The two postulates of relativity theory also lead to the relativity of a second fundamental measured quantity, length. Einstein again applied the two postulates to a thought experiment (not a real experiment) on a simple measuring process. This was one way of deducing the physical consequences from his two fundamental postulates. Again the constant speed of light is the key, while the relativity principle is the underlying assumption.

We'll give Jane and John a rest and ask Alice and Alex, two other virtual researchers, to perform this thought experiment. Let Alice be at rest, while Alex is riding on a platform moving at uniform velocity relative to her. Alex carries a meter stick to measure the length of his platform in the direction it is moving. He obtains exactly 1 m. Alice tries to measure the length of Alex's platform with her meter stick as Alex's platform moves past her at constant velocity. She has to be quick, since she must read the two ends of the meter stick at the exact same instant; otherwise if she measures one end first, the other end will have moved forward before she gets to it. But there is a problem: light from the front and the rear of the platform take a certain amount of time to reach her, and in that brief lapse of time, the platform has moved forward.

Using only a little algebra and an ingenious argument (see the insert "Length Contraction"), Einstein derived an equation relating the measurements made by our two observers. The calculation, which is similar to the one for time dilation, yielded the result that, because the speed of light

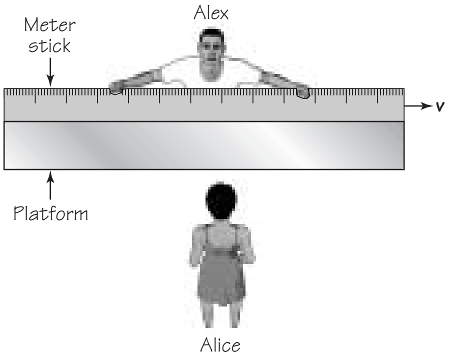


FIGURE 9.11 Length and contraction of a meter stick.

is not infinite, Alice's measurement of the length of the moving platform always turns out to be shorter than the length that Alex measures. The faster the platform moves past her, the shorter it is by Alice's measurement. The lengths as measured by the two observers are related to each other by the same square root as for time dilation. Alex, who is at rest relative to his platform, measures the length of the platform to be l_s, but Alice, who must measure the length of Alex's moving platform from her stationary frame, measures its length to be l_m. Einstein showed that, because of the constant speed of light, these two lengths are not equal but are related instead by the expression

$$l_m = l_s \sqrt{1 - \frac{v^2}{c^2}}.$$

Again the square root appears, which is now multiplied by the length l_s in Alex's system to obtain the length l_m as measured by Alice. Again, you will notice that when $v = 0$, i.e., when both systems are at rest with respect to each other, the equation shows there is no difference between l_m and l_s, as we expect. When the platform moves at any speed up to nearly the speed of light, the square root becomes a fraction with the value less than 1, which indicates that l_m is less than l_s. The conclusion:

> Length measurements are not absolute and unchanging, but relative. In fact, an object moving relative to a stationary observer appears to that observer in that reference frame to be shorter in the direction of motion than when its length is measured by an observer moving with the object—and it appears shorter the faster the object is moving.

LENGTH CONTRACTION

Consider a meter stick in a spaceship moving past you at high speed v. The meter stick is aligned in the direction of motion. Alex is an observer riding on the spaceship. He has a high-speed timing device and a laser emitter. With that equipment, she intends to measure the speed of light by emitting a laser pulse along a meter stick, which is aligned along the direction of motion of his spaceship. He will time the duration required for the light pulse to traverse the length of the meter stick. After performing the measurement, the time interval he measures is T_s and the length of the meter stick is l_s, the s indicating that they are stationary relative to her. Calculating the speed of the light pulse, l_s/T_s, he obtains the speed of light c, as expected.

Meanwhile, Alice is fixed on Earth as Alex's spaceship speeds past. She observes his experiment and makes the same measurements using her own clock—however her result for the time interval T_m registered on Alex's moving clock is different from Alex's measurement because of time dilation. Nevertheless, according to Einstein's second postulate Alice must obtain the exact same value for the speed of the light pulse, c. The only way this is possible is if the length of the meter stick in Alex's moving spaceship as measured by Alice, l_m as measured with her own measuring device, appears to have contracted by the same amount that the time interval she measured on the moving clock has expanded. The moving length l_m must therefore appear to be contracted in the direction of motion according to the relationship

$$l_m = l_s\sqrt{1 - \frac{v^2}{c^2}}.$$

This effect is known as *length contraction*. But note that the object is not actually contracting as it moves—the observed "contraction," which is in the direction of motion only, not perpendicular to it, is an effect of the *measurement* made from another system—as was the effect on the relative observations of elapsed time, the "time delay."

When $v = 0.8c$, for example, the apparent foreshortening seen by Alice of Alex's platform moving to the right, and of Alex himself and everything moving with him, would be about $0.6\ l_s$. Moreover, it is symmetrical! Since Alex can consider his frame to be at rest, Alice seems to be moving fast to the left, and it is she and her platform which seem to Alex to be foreshortened by the same amount.

The apparent contraction continues all the way up to the speed of light, at which point the length of the moving object would appear to the stationary observer to be zero. However, no mass can be made to reach the speed of light, so we can never attain zero length, although in accelerators (colliders) elementary particles come pretty close to that limit.

9.9 RELATIVITY OF MASS

You saw in Section 3.4 that inertial mass is the property of objects that resists acceleration when a force is applied. The inertial mass, or simply "the mass," is the constant of proportionality between force and acceleration in Newton's second law of motion

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

Therefore a constant force will produce a constant acceleration. So, once an object is moving, if you keep pushing on it with the same force, it will keep accelerating, going faster and faster and faster without limit, according to this formula. Newton's second law thus contains no speed limit. But this is inconsistent with the relativity theory, which imposes a speed limit for objects in space of about 300,000 km/s (186,000 mi/s), the speed of light. The way out is to amend Newton's second law. Einstein's way was to note that m, the inertial mass, does not stay constant but increases as the speed increases—as in fact is experimentally observed, for example, for high-speed elementary particles. When the speed increases, it takes more and more force to continue the same acceleration—eventually an infinite force trying to reach the speed of light. Einstein deduced from the two postulates of special relativity theory that the inertia of a moving object increases with speed, and it does so in the same way as the time relation in time dilation. (The derivation is provided in the *Student Guide* for this chapter.) Using our familiar square root factor, we can write

$$m_{\text{m}} = \frac{m_{\text{s}}}{\sqrt{1 - v^2/c^2}}.$$

Here m_{m} is the mass of the object in relative motion, and m_{s} is the mass of the same object before it starts to move. Often m_{s} is called the "rest mass."

Similarly to the measurement of time intervals, as an object's speed increases the mass as observed from a stationary reference frame also increases. It would reach an infinite (or undefined) mass if it reached the speed of light. This is another reason why anything possessing mass cannot actually be made to attain the speed of light; it would require applying an infinite force to accelerate it to that speed. By the same argument, entities that do move at the speed of light, such as light itself, must therefore have zero rest mass.

Following Einstein's result that the mass of an object increases when it is in motion relative to a stationary observer, Newton's equation relating the force and the acceleration can be written as a more general law

$$\mathbf{F}_{\text{net}} = \frac{m_{\text{s}}}{\sqrt{1 - v^2/c^2}}\,\mathbf{a}.$$

THE RELATIVISTIC INCREASE OF MASS WITH SPEED

v/c	m/m_0	v/c	m/m_0
0.0	1.000	0.95	3.203
0.01	1.000	0.98	5.025
0.10	1.005	0.99	7.089
0.50	1.155	0.998	15.82
0.75	1.538	0.999	22.37
0.80	1.667	0.9999	70.72
0.90	2.294	0.99999	223.6

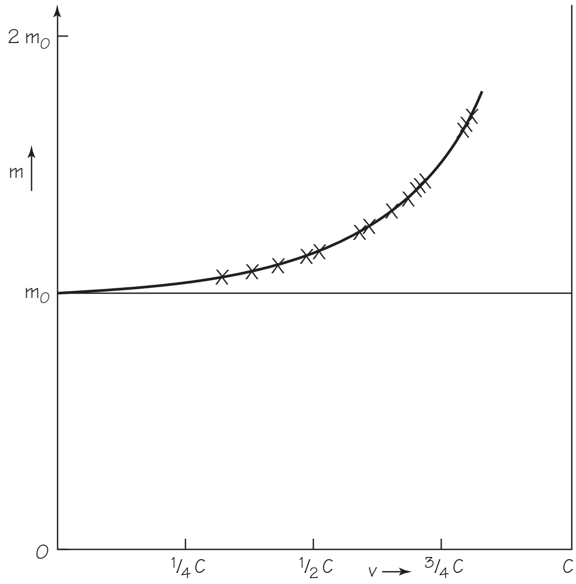


FIGURE 9.12 The increase of mass with speed. Note that the increase does not become large until v/c well exceeds 0.50.

Notice that as the relative speed decreases to zero, this equation transforms continuously into Newton's equation

$$\mathbf{F}_{\text{net}} = \frac{m_{\text{s}}}{\sqrt{1 - v^2/c^2}}\,\mathbf{a} \rightarrow \mathbf{F} = m\mathbf{a}, \text{ as } v \rightarrow 0.$$

This indicates again that Einstein's physics did not break with Newton's physics. Instead Einstein's physics is a continuation of Newton's physics.

9.10 MASS AND ENERGY

After Einstein completed his paper on the special theory of relativity in 1905 he discovered one more consequence of the relativity postulates, which he presented, essentially as an afterthought, in a three-page paper later that year. In terms of the effect of physics on world history, it turned out to be the most significant of all his findings.

We discussed in Chapter 5 that when work is done on an object, say hitting a tennis ball with a racket, the object acquires energy. In relativity theory, the increase in speed, and hence the increase in kinetic energy of a tennis ball or any object, results in an increase in mass (or inertia), although in everyday cases it may be only an infinitesimal increase.

Examining this relation between relative speed and effective mass more closely, Einstein discovered that *any* increase in the energy of an object should yield an increase of its measured mass—whether speeding up the object, or heating it, or charging it with electricity, or merely by doing work by raising it up in the Earth's gravitational field. In short, Einstein discovered that a change in energy is equivalent to a change in mass. Moreover, he found that the equivalence works both ways: An increase or decrease in the energy in a system correspondingly increases or decreases its mass, and an increase or decrease in mass corresponds to an increase or decrease in energy. In other words, mass itself is a measure of an equivalent amount of energy.

To put Einstein's result in symbols and using the delta (Δ) symbol: a change in the amount of energy of an object is directly proportional to a change in its mass, or

$$\Delta E \propto \Delta m.$$

Einstein found that the proportionality constant is just the square of the speed of light, c^2:

$$\Delta E = (\Delta m)c^2,$$

or, expressed more generally,

$$\boxed{E = mc^2.}$$

In its two forms, this is probably the most famous equation ever written. What it means is that an observed change of mass is equivalent to a change of energy, and vice versa. It also means that an object's mass itself, even if it doesn't change, is equivalent to an enormous amount of energy, since the proportionality constant, c^2, the square of the speed of light in vacuum, is a very large number. For example, the amount of energy contained in just 1 g of matter is

$$\begin{aligned} E = mc^2 &= (0.001\ \text{kg})(3 \times 10^8\ \text{m/s})^2 \\ &= (1 \times 10^{-3}\ \text{kg})(9 \times 10^{16}\ \text{m}^2/\text{s}^2) \\ &= 9 \times 10^{13}\ \text{kg m}^2/\text{s}^2 \\ &= 9 \times 10^{13}\ \text{J}. \end{aligned}$$

This enormous amount of energy is roughly equivalent to the chemical energy released in 20 tons of TNT, or the amount of energy consumed in the whole United States on average in 30 s. It is the source of the energies released by radioactive substances, our Sun and other stars, by nuclear weapons, and by nuclear reactors producing electrical energy.

Not only are mass and energy "equivalent," we may say *mass is energy*. This is just what Einstein concluded in 1905: "The mass of a body is a measure of its energy content." We can think of mass as "frozen energy," frozen at the time the Universe cooled soon after the Big Bang and energy clumped together into balls of matter, the elementary particles of which ordinary matter is made. Thus any further energy pumped into a mass will increase its mass even more. For instance, as we accelerate protons in the laboratory to nearly the speed of light, their mass increases according to the relativistic formula for m_m. This increase can also be interpreted as an increase in the energy content of the protons. These two different deductions of relativity theory—mass increase and energy–mass equivalence—are consistent with each other.

This equivalence has exciting significance. First, two great conservation laws become alternate statements of a single law. In any system whose total mass is conserved, the total energy is also conserved. Second, the idea arises that some of the rest energy might be transformed into a more familiar form of energy. Since the energy equivalent of mass is so great, a very small reduction in rest mass would release a tremendous amount of energy, for example, kinetic energy or electromagnetic energy.

9.11 CONFIRMING RELATIVITY

Einstein's theory is not only elegant and simple, it is extraordinarily far-reaching, although its consequences were and still are surprising when first encountered. By noticing an inconsistency in the usual understanding of

Note: Einstein did not initially call his theory the theory of relativity. That term was given to it by others. Einstein later said he would have preferred calling it the theory of *invariance*. Why? Because, as said before, the laws of physics remain invariant, unchanged, the same for the "stationary" and the "moving" observer. That is extremely important, and makes it obvious why it is so *wrong* to say that Einstein showed that "everything is relative."

Maxwell's theory, and by generalizing Galileo's ideas on relative motion in mechanics, Einstein had been led to state two general postulates. Then he applied these two postulates to a study of the procedures for measuring the most fundamental concepts in physical science—time, length, mass, energy—and, as one does in a geometry proof, he followed these postulates to wherever the logic led him. The logic led him to conclude that the measurements of these quantities can be different for different observers in motion relative to each other. While the laws of physics—properly amended, as in the case of $\mathbf{F}_{\text{net}} = m\mathbf{a}$ becoming $\mathbf{F}_{\text{net}} = (m_s/\sqrt{1 - v^2/c^2})\,\mathbf{a}$—and the speed of light are the same for all observers, these basic quantities that enter into the laws of physics, such as time or mass, are not the same for all, they are relative with respect to the measurement frame. This is why it is called the theory of relativity. More precisely, it is called the theory of *special* relativity, since in this theory the relative velocities of the observers must be uniform (no acceleration), hence applying only to inertial systems.

But, you may object, anyone can come up with a couple of postulates, correctly deduce some strange consequences from them, and claim that they now have a new theory. In fact, this happens all too often, and usually is rejected as poor science. Why do we accept Einstein's theory as good science? The answer is of course eventual experimental confirmation, internal consistency, and consistency with other well-established theories. Every theory in science, whether deduced from a few postulates or induced from experimentally based hypotheses, must pass the rigorous test of experimental examination by various researchers, usually over a long period of time. In fact, as one astronomer recently remarked, the more profound the theory, the more extensive the experimental evidence that is required before it can be accepted. In addition, of course, the derivation of the theory cannot contain any logical mistakes or unfounded violations of accepted laws and principles. And it must be compatible with existing theories, or else it must show how and why these theories must be revised.

Far from being "dogmatic," as some would have it, scientists are always skeptical until the evidence is overwhelming. Indeed, it took more than a decade of research to confirm that relativity theory is indeed internally consistent as well as experimentally sound. The above sections also indicate how and why the classical physics of Newton and Maxwell had to be revised for application to phenomena at high relative speeds. But as the relative speed decreases, all of the results of relativity theory fade smoothly into the classical physics of the everyday world. There is no "incommensurability" between the worlds of Newton and Einstein.

Relativity theory is so well tested that it is now used as a tool for studying related theories and for constructing new experiments. Most of these experiments involve sub-microscopic particles moving at extremely high speeds, such as are found in modern-day accelerators. But some are also at everyday speeds. Here are a few of the most well-known confirmations of the postulates and deductions of special relativity theory.

The Constancy of the Speed of Light

The validity of the two postulates of relativity theory also extends to classical physics (e.g., mechanics), as Galileo showed for the early relativity postulate with the tower experiment, and as Einstein apparently realized as he thought about running alongside a light beam. A direct confirmation of the constancy of the speed of light has been obtained from the study of double stars, which are stars that orbit about each other. If the orbit of one star is close to the line of sight from the Earth, then at one side of the orbit it is moving toward the Earth, on the other side it is moving away. Careful studies of the speed of light emitted by such stars as they move toward and away from us at high speed show no difference in the speed of light, confirming that the speed of light is indeed independent of the speed of the source.

Another of the many experiments involved a high-speed particle in an accelerator. While moving at close to the speed of light, it emitted electromagnetic radiation in opposite directions, to the front and to the rear. Sensitive instruments detected the radiation and measured its speed. Astonishing as it may seem to the uninitiated, the speed of the radiation emit-

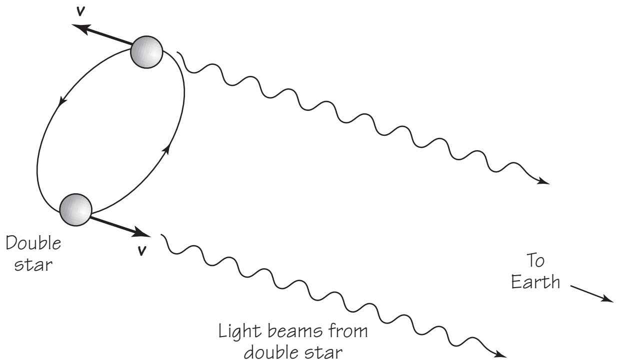


FIGURE 9.13 Light beams from a double-star system.

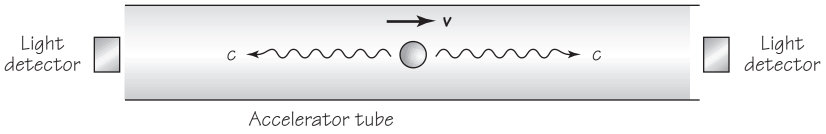


FIGURE 9.14 Particle in an accelerator emitting light beams simultaneously in opposite directions.

ted in both directions turned out to be exactly the speed of light, even though the particle itself was moving close to the speed of light—a striking confirmation of the constant-light-speed postulate, which amounts to a law of nature.

The Relativity of Time

The relativity theory predicts that a moving clock, as seen by a stationary observer, will tick slower than a stationary clock. We noted earlier that this effect has been tested and confirmed using atomic clocks inside airplanes and satellites.

An equally dramatic confirmation of the relativity of time occurred with the solution to a curious puzzle. Cosmic rays are high-speed protons, nuclei, and other particles that stream through space from the Sun and the galaxy. When they strike the Earth's atmosphere, their energy and mass are converted into other elementary particles—a confirmation in itself of the mass–energy equivalence. One of the particles they produce in the atmosphere is the so-called *mu-meson*, or simply the *muon*. When produced in the laboratory, slow muons are found to have a short life. On average they last only about $2.2\ 10^{-6}$ s, at which time there is a 50–50 chance that each one will decay into other elementary particles. (10^{-6} s is a microsecond, symbol: μs.)

The puzzle is that the muons created in the upper atmosphere and moving at high speed were found to "live" longer before they decay than those laboratory-generated ones. They last so long that many more survive the long trip down to the detectors on the ground than should be possible. Considering the speed they are traveling and the distance they have to traverse from the upper atmosphere to sea-level (about 30 km), their average lifetime of 2.2 μs, as measured for slow muons, should not be sufficient for them to survive the journey. Most of them should decay before hitting the ground; but in fact most of them do reach the ground. How can this be? The answer is the time dilation predicted by relativity theory. Relative to the detectors on the ground, the muons are moving at such high speed that their "clock" appears slowed, allowing them to survive long enough to reach the ground. The amount of slowing, as indicated by the number of muons reaching the ground, was found to be exactly the amount predicted by relativity theory.

Relativity of Length

Recall that one of the basic ideas of relativity theory is that all speed is relative to the observer who measures the speed. So let's return to the puzzle of the long-lived muons, only this time we will jump to the perspective of the hapless muon crashing through the Earth's atmosphere. From the muon's point of view, it is at rest, while the Earth is flying up toward it at close to the speed of light. Since the Earth is now moving with respect to the muon's own frame of reference, the distance from the top of the atmosphere to the ground undergoes a length contraction when seen from the point of view of the muon. The contraction is so great that, from the muon's perspective, it has no trouble covering this short distance in the mere 2.2 μs of the short life it has in its "stationary" reference frame. Again, the observations are in complete agreement with the predictions of special relativity theory.

Relativity of Mass

Relativity theory predicts that the observed mass of an object will increase as the relative speed of the object increases. Interestingly, this effect had been observed even before Einstein's theory, when scientists were puzzled to notice an increase in the mass of high-speed electrons in vacuum tubes. This effect is easily observed today in particle accelerators, where elementary charged particles such as electrons or protons are accelerated by electromagnetic fields to speeds as high as 0.9999999 the speed of light. The masses of these particles increase by exactly the amount predicted by Einstein's formula. At that speed the increase of their mass (m_{m} is about 2236 times the rest mass; $m_{\mathrm{m}} = 2236\ m_{\mathrm{s}}$). In fact, circular accelerators have to be designed to take the mass increase into account. As the particles are accelerated to high speeds by electric fields, they are curved into a circular path by magnetic fields to bring them back and let them undergo repeated accelerations by the fields. You saw in Section 3.12 that an object moving in a circular path requires a centripetal force. This force is given by the equation $F = mv^2/R$. Here R is the radius of the circle, which is fixed; v is the particle's speed, which increases; and m is the moving mass, which also increases according to relativity theory.

If scientists do not take the mass increase into account in their particle

FIGURE 9.15 Fermi National Accelerator Laboratory (Fermilab), in Batavia, Illinois, one of the world's most powerful particle accelerators.

accelerators, the magnetic force would not be enough to keep the particles on the circular track of the accelerator, and they would hit the wall or come out there through a portal. A simple circular accelerator is called a *cyclotron*. But when the increase of the accelerating force is precisely synchronized with the increases in speed and relativistic mass, it is called a *synchrocyclotron*.

Equivalence of Mass and Energy

Einstein regarded the equivalence of mass and energy, as expressed in the equation $E = mc^2$, to be a significant theoretical result of special relativity, but he did not believe it had any practical importance when he announced his finding. The hidden power became most obvious, of course, in the explosion of the atomic (more precisely "nuclear") bombs in 1945. The tremendous energy unleashed in such a bomb is derived from the transformation in the nuclei of a small amount of uranium or plutonium mass into the equivalent, huge amount of energy.

FIGURE 9.16 Trajectories of a burst of elementary particles in the magnetic field inside a buble chamber.

Nuclear bombs and reactors are powered by the splitting of heavy atoms. An opposite process, a fusion reaction, takes place using the joining together of nuclei of light elements. Again, a tiny amount of the mass is converted into energy according to Einstein's formula. Despite much effort, it has not yet been possible to control this fusion process on a scale sufficient to produce electricity for domestic and industrial use; however, the absence of harmful radioactive by-products would make such a device very desirable. But the nuclear fusion process does have a very practical importance: It powers the energy output of the Sun and all other stars in the Universe. Without it, life could not exist on the surface of the Earth. (Nuclear fission and fusion and their applications are further discussed in Chapter 18.)

The conversion of energy into mass can also be observed in the collisions of elementary particles that have been accelerated to enormously high speeds. Photographs of the results, such as the one here, display the creation of new particles.

9.12 BREAKING WITH THE PAST

Although Einstein's theory of special relativity did not represent a major break with classical physics, it did break with the mechanical world view. Our understanding of nature provided by special relativity, together with subsequent advances in quantum mechanics, general relativity theory, and other innovations, will slowly shape the new world view that is emerging.

Special relativity introduced an important break with the mechanical world view concerning the notion of absolute rest and absolute motion, which ceased to exist as a result of Einstein's work. Until that time, most physicists defined absolute rest and motion in terms of the so-called ether, the stuff that filled all of the space and transmitted light and electric and magnetic forces. As noted earlier, Einstein simply ignored the ether as "superfluous," since only relative motions were used in his theory. At the same time, and even before, a large number of careful experiments of different sorts to detect the ether had failed completely. One of these, the most famous one, was a series of experiments, during the 1880s, in which the American scientists Albert A. Michelson and Edward Morley attempted to detect the "wind" of ether experienced by the Earth as it moved through the supposed stationary ether on its orbit around the Sun. If such an ether existed, scientists believed, it should cause an "ether wind" over the surface of the Earth along the direction of motion. Since light was believed to be a wave moving through the ether, somewhat like sound waves through the air, it should be affected by this wind. In particular, a light wave traveling into the wind and back should take longer to make a round trip than a wave traveling the exact same distance at a right angle, that is, across the wind and back. (See the calculation in the *Student Guide*.) Comparing two such waves, Michelson and Morley could find no difference in their times of travel, within the limits of precision of their experiment. Within a few years of Einstein's theory, most physicists had abandoned the notion of an ether. If it could not be detected, why keep it?

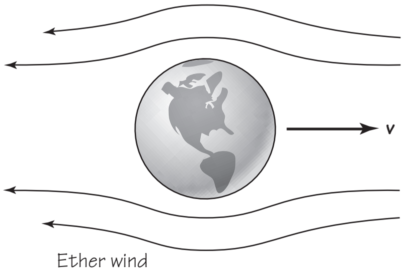


FIGURE 9.17 Earth moving through the stationary ether, according to nineteenth-century concepts.

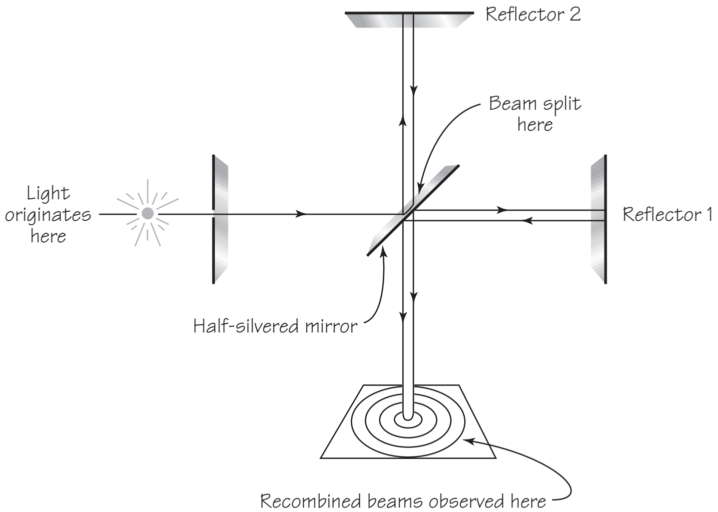


FIGURE 9.18 Schematic diagram of the Michelson–Morley experiment.

Not only did the loss of the ether rule out the concepts of absolute rest and absolute motion, but scientists had to rethink their understanding of how forces, such as electricity, magnetism, and gravity, operate. Ether was supposed to transmit these forces. Suddenly there was no ether; so what are these fields? Scientists finally accepted the idea that fields are independent of matter. There was now more to the world than just matter in motion. There were now matter, fields, and motion, which meant that not everything can be reduced to material interactions and Newton's laws. Nonmaterial fields had also to be included, and be able to carry energy across empty space in the form of light beams. The world was suddenly more complicated than just matter and motion. (You will read more about fields in Chapter 10.)

Another break with the mechanical world view concerned the concepts of space and time. Newton and his followers in the mechanical view had regarded space and time to be absolute, meaning the same for all observers, regardless of their relative motion. Einstein demonstrated in special relativity that measurements of space and time depend upon the relative motion of the observers. Moreover, it turned out that space and time are actually entwined with each other. You can already see this in the problem of making measurements of the length of a moving platform. The ends of the meter stick must be read off at the ends of the platform at the same instant in time. Because of the postulate of the constancy of the speed of light, a person at rest on the platform and a person who sees the platform moving will not agree on when the measurements are simultaneous. In 1908 the German mathematician Hermann Minkowski suggested that in relativity theory time and space can be viewed as joined together to form the four dimensions of a universal four-dimensional world, called *spacetime*. Four-dimensional spacetime is universal because an "interval" measured in this world would turn out to be the same for all observers, regardless of their relative motion at uniform velocity, but the "interval" would include both distance and time.

The space in which we live consists, of course, of three dimensions: length, width, and breadth. For instance, the event of a person sitting down on a chair in a room can be defined, in part, by the person's three coordinates. Starting at one corner of the room, the length along one wall may be 3 m, the length along the other wall may be 4 m, and the height to his chair seat may be 0.5 m. But to specify this event fully, you must also specify the time: say, 10:23 a.m. These four coordinates, three of space and one of time, form the four dimensions of an event in *spacetime*. Events take place not only in space but also in time. In the mechanical world view, space and time are the same for all observers and completely independent of each other. But in relativity theory, space and time are different for different observers moving relative to each other, and space and time are entwined together into a four-dimensional construct, "spacetime," which is the same for all observers.

SOME NEW CONCEPTS AND IDEAS

constructive theory
ether
length contraction
mass–energy equivalence
Michelson–Morley experiment
principle of constancy of speed of light
principle of relativity
reference frame
reference frame, inertial
relative motion
spacetime
theory of special relativity
time dilation

FURTHER READING

D. Cassidy, *Einstein and Our World* (Amherst, NY: Prometheus Books, 1995).
A. Einstein, *Ideas and Opinions* (New York: Bonanza Books, 1988).

A. Einstein, *Relativity: The Special and the General Theory* (New York: Crown, 1995), and many other editions; originally published 1917.
A. Einstein, *The World As I See It* (New York: Citadel Press, 1993).
A. Einstein and Leopold Infeld, *The Evolution of Physics* (New York: Simon and Schuster, 1967).
A. Fölsing, *Albert Einstein: A Biography*, E. Osers, transl. (New York: Penguin, 1998).
P. Frank, *Einstein: His Life and Times*, rev. ed. (New York: Da Capo).
M. Gardner, *Relativity Simply Explained* (New York: Dover, 1997).
B. Hoffmann, *Albert Einstein: Creator and Rebel* (New York: Viking Press, 1972).
G. Holton, *Einstein, History, and Other Passions* (Cambridge, MA: Harvard University Press, 2000).
G. Holton, *Thematic Origins of Scientific Thought: Kepler to Einstein* (Cambridge, MA: Harvard University Press, 1988), Part II: "On Relativity Theory."
G. Holton and S.G. Brush, *Physics, the Human Adventure*. (Piscataway, NJ: Rutgers University Press, 2001), Chapter 30.
E.F. Taylor, and J.A. Wheeler, *Spacetime Physics: Introduction to Special Relativity*, 2nd ed. (New York: Freeman, 1992).
H. Woolf, ed., *Some Strangeness in the Proportion: A Centennial Symposium to Celebrate the Achievements of Albert Einstein* (Reading, MA: Addison-Wesley, 1980).

Web sites

See the course Web site at: http://www.springer-ny.com/
A. Einstein: http://www.aip.org/history/einstein
A. Einstein: http://www.pbs.org/wgbh/nova/einstein

STUDY GUIDE QUESTIONS

1. What is "relative" in the theory of relativity?
2. What is special about the theory of special relativity?
3. Why did Einstein later say he would have preferred if it had been called the theory of invariance (or constancy)?
4. State in your own words the two principles, or postulates, on which special relativity is based.
5. What are four deductions of the theory?
6. Briefly describe an experimental confirmation of each of these four deductions.

9.1 The New Physics

1. What is "classical physics"?
2. What was new about the new physics?

9.2 Albert Einstein

1. Who was Albert Einstein? What did he do?
2. Give a brief synopsis of his life. Give a brief synopsis of his views apart from science.

9.3 The Relativity Principle

1. How does Einstein's theory compare with other theories described in previous chapters?
2. What is relative motion? Give an example.
3. A student measures the speed of a cart on a laboratory table to be 150 cm/s toward the north. The laboratory table is on wheels and is moving forward at 150 cm/s to the north. What is the speed of the cart relative to the floor?
4. What do Einstein's Relativity Principle and the Galilean Relativity Principle have in common? In what ways are they different?
5. A sailing ship is moving at uniform velocity on a calm sea. A ball drops from the mast of the ship.
   (a) Where does the ball land?
   (b) Compare the observation of the falling ball by a person riding on the ship with the observation of another person standing on the shore. Explain any difference.
   (c) If these are different, then which observation is correct?
6. Can the observer on the ship that is moving smoothly forward determine (from a windowless cabin within) if he is really moving or not? Explain.
7. Two people moving relative to each other while observing a falling ball see two different trajectories. Why would they be wrong to say as a result that "everything is relative"?
8. What does the statement mean that physical measurements, but not physical laws, depend on the observer's frame of reference?

9.4 Constancy of the Speed of Light

1. A small asteroid is headed straight toward the Earth at 20 km/s. Suddenly a gas jet on the asteroid fires a chunk of rock toward the Earth with a speed of 3 km/s relative to the asteroid. Scientists on Earth measure the speed of the chunk as it flies toward them. What is the measured speed?
2. An elementary particle is moving toward the Earth at 0.999999 the speed of light. It emits a light wave straight at the Earth. The light wave is detected by equipment on the ground and the speed of the wave is measured. What will the measured speed be? With what speed does the light move away from the elementary particle, as measured by an experimenter moving along with the particle?

9.5–9.7 Relativity of Time and Time Dilation

1. In your own words explain the thought experiment that shows that to an observer who is stationary, a moving clock consisting of a laser and mirror runs slower than it does to an observer riding with the clock.
2. Examine the formula for time dilation and define every symbol in it.
3. Using the formula for time dilation, explain why a stationary observer will measure that time slows down for events in the moving system, for relative speeds greater than zero but less than c.
4. The equation for time dilation refers to any relative speed v up to the speed of light c.
   (a) What happens when $v = 0$?
   (b) Why is it that we don't notice any time dilation at even the fastest speeds we can encounter in the everyday world, for example, a supersonic jet plane?
5. Explain why the time dilation equation indicates that there can be no relative speeds of objects greater than the speed of light.

9.8–9.9 Relativity of Length and Mass

1. In your own words present the thought experiment that shows that to an observer who is stationary, a moving meter stick appears shorter than it does to an observer riding with the meter stick.
2. Examine the formula for length contraction and define every symbol in it.
3. Using the formula for length contraction, show what happens when the meter stick is moving at 260,000 km/s and its length is measured by an observer at rest.
4. Why would it take an infinite force to accelerate any mass up to the speed of light?
5. Why can't anything that possesses mass at zero speed attain the speed of light?
6. Why is it wrong to say that Einstein's relativity represents a sharp break with Newtonian mechanics?
7. Why is it wrong to say that time is really dilated and length is really contracted in a moving system?
8. Why has it been wrong to say, as some did, that relativity theory undermines morality because it shows that "it all depends on your point of view"?

9.10 Mass and Energy

1. What does it mean to say that mass and energy are equivalent?
2. Illogically, one often hears that mass can be "converted" into energy, and energy be "converted" into mass. What would be a more accurate way of expressing the facts summarized by $E = mc^2$?
3. If mass and energy are interchangeable, what happens to the law of conservation of energy?

9.11 Confirming Relativity

1. By using different frames of reference, give two explanations for the fact that numerous high-speed muons generated in the upper atmosphere are able to

survive the trip to the Earth's surface, even though they are so short-lived that only a few should survive.

2. Aside from the increasing speed of the elementary particles, why does the centripetal force in a circular accelerator have to be constantly increased as a bunch of particles moves faster and faster?

9.12 Breaking with the Past

1. In what ways did relativity theory introduce a sharp break with the mechanical world view?
2. How were electric forces, fields, and the ether believed to be related?
3. What was the Michelson–Morley experiment? What were they attempting to detect? How did they attempt to detect it? What was their result?
4. What impact on the mechanical view did the rejection of the ether have?
5. What other break with the past did special relativity introduce?
6. What is four-dimensional spacetime? Give an example of an event in four-dimensional spacetime.

DISCOVERY QUESTIONS

1. Two observers are in uniform relative motion with respect to each other. They are in direct communication by cell phone and are attempting to decide who is really in motion and who is really at rest. What arguments can each one give to claim that he is at rest and the other person is moving?
2. Some people have argued that the theory of relativity supports the idea that "everything is relative." Would you agree or disagree with this conclusion, and how would you support your position, say in a discussion with others in class?
3. A person is exercising on a tread mill in a gym. The speed of the tread mill is set at 3 mi/hr. What is the speed of the exerciser relative to the ground? What is the exerciser's speed relative to the belt of the tread mill?
4. In studying this Part One of the text, you have followed some of the immense transformation of humankind's culture, from the pre-scientific period to current research questions. After thoughtful reflection on this experience, write a page or two summarizing the stages in this adventure of the creative mind.

Quantitative

5. On a piece of graph paper, plot the results, for a few distributed points, of the equation for length contraction, with l_{m} on the y-axis and the ratio v/c on the x-axis from 0 to 1.
   (a) Study this graph and explain why using this relativistic equation is usually not needed for relative speeds below about 20% of the speed of light.
   (b) From this graph explain why objects cannot attain the speed of light.
   (c) Explain why objects cannot go faster than the speed of light.

6. How fast would a pitcher have to throw a baseball to have its mass increase by 1%?
7. How much would be the rate of your body's "clock," the heart beat, decrease as measured by someone at rest on the ground, if you were flying in a plane at the speed of sound, about 330 m/s?
8. In Question 7, if you weighed yourself on a supersensitive scale during flight, how much would your weight seem to have increased, if at all, as observed from the ground?
9. If you have an opportunity to use the computer program "Space-Time," use the program to take a "trip" to Alpha Centauri, and observe the twin paradox. How is the paradox resolved?